NONLINEAR EFFECTS ON FOCUSSED WATER WAVES

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Abstract

A brief account is given of theory and experiments for water wave focussing. The theory uses weakly nonlinear wave modulation theory, that is the nonlinear Schrödinger equation, summarises earlier theoretical papers and augments them with numerical results. Experiments were performed to compare with theory. The limited comparison shown here indicates that the theory gives satisfactory results even for waves close to breaking. Both the numerical and experimental results indicate the importance of linear diffraction when waves are focussed. The relevance of diffraction is easily assessed, and is likely to dominate in many coastal examples of weak focussing.

Introduction

In practical examples of water-wave refraction, ray diagrams frequently show rays crossing. Often the initial ray intersections involve only a few rays with no well defined structure apparent. Any crossing of rays is an indication that the ray-theory approximations have become invalid, and a greater density of rays will normally reveal a focussing of rays near the initial crossing point. In practice simple smoothing or averaging methods are sometimes used to calculate wave heights. Where these have no rational basis there is always an uncertainty about possible steep waves due to focussing being missed.

Diffraction effects need to be included in order to obtain accurate solutions near a focus. In addition, if waves are steep nonlinear effects are significant. This paper reports on theoretical and experimental work.

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Model examples

In order to clarify ideas we consider two simple cases. In each case we suppose that waves have already suffered refraction due to propagation over a shoaling area or through non uniform currents which focus them but that the waves now propagate toward a focus over water of uniform depth. The focussing portion of the wave front is taken to be circular subtending an angle 2α at the focus which is at a distance f. The rest of the wave front is taken to be straight. In case (a) waves away from the focussing region are taken to be parallel to the central ray through the focus, and in case (b) they are taken to be smooth straight extensions of the circular arc; see figure 1.

In case (a) there are no further ray crossings until well beyond the focus and study can be directed to wave amplitudes in the focal region. In case (b) waves from both sides continue to cross behind the focus and wave amplitudes can be expected to be at least twice those of the original wave train, and any exception near the focus is of interest.

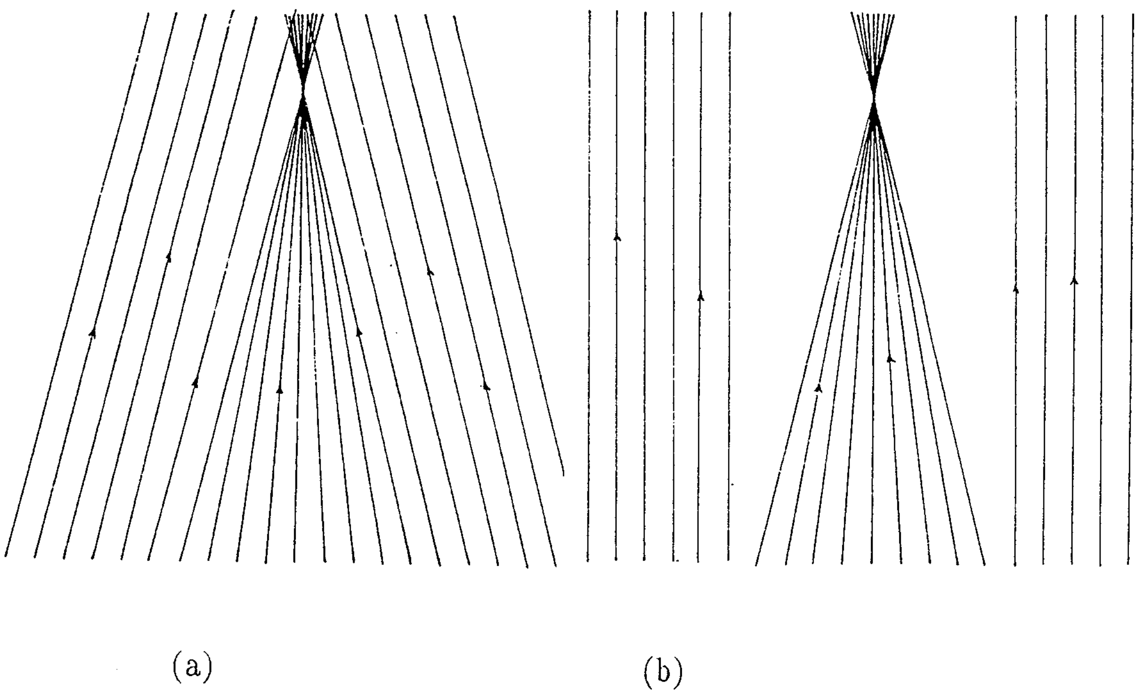

Figure 1 Two examples of wave focussing (a) where the focussing region is not influenced by the wave front on each side of the initial focussing wave. (b) where the waves each side of the focussing wave meet close to the focussing region.

Linear waves

Since solutions of the linear wave equation may be superposed, the initial conditions may be broken down into sub-problems: two semi-infinite plane waves with their attendant diffraction and the focussing arc. The latter is of the greater interest since the half-plane waves correspond to Sommerfeld's solution for diffraction by a semi-infinite barrier. Exact solutions of the linear problem are easily created by adding plane-wave solutions. We have used one particular example to represent a focussing arc. It is a set of N plane waves of equal amplitude and equal angular spacing in the range $(-\alpha,\alpha)$ all in phase at the focus. We call this example F and it is taken to be illustrative of the case shown in figure 1(a).

It is easy to compute solutions for example F corresponding to the focus having differing angles 2α of incoming waves and different distances, f, from an initial line. The effects of diffraction show for $\alpha = 15°$ and a focal distance f = 10L, (L is wavelength), this exact solution shows an increase of amplitude over the initial conditions of only 20%. This small increase is due to diffraction counter balancing the focussing effects. On the other hand initial conditions with the same value of α but larger values of f do lead to focussing with enhancement of wave amplitude, so that it might approach the estimate of $2\alpha(2f/L)^{\frac{1}{2}}$ given, in different notation, by Peregrine (1986).

The effects of diffraction scale with the Fresnel number

$$N = \alpha^2 f/L.$$

The example given above, with N = 0.7, is chosen since it represents a balance between diffraction and focussing. For large N, focussing is dominant. In any specific example it is easy to calculate N and assess diffraction. For example, if waves have a wavelength of 80 metres and part of the wave front of angle 20°, i.e. $\alpha = 10° = 0.174$ radius, focusses at a distance of 2000 metres, then N = 0.76, which implies strong diffractive effects

Nonlinear waves

A major effect of nonlinearity on refraction is wave defocussing. Some aspects of this are described in Peregrine (1983, 1985 and 1986) for examples where Stokes's wave theory is a good approximation for periodic waves, i.e. not too shallow water. From a practical point of view this defocussing is reassuring since the nonlinear effects are reducing maximum amplitudes. However, unlike linear waves, theory shows that the effects of the waves bounding the waves that focus cannot be treated by simple superposition of solutions.

For the configuration of figure 1(a) the bounding waves do not contribute significantly more energy to the focussing region, in fact they act to spread the disturbance of the wave front more rapidly, since nonlinear effects lead to a "splitting" of linear rays (Peregrine, 1983).

More care is needed when assessing the effects of converging waves such as in figure 1(b). Here linear theory indicates that behind the focussing region the bounding waves contribute twice the initial amplitude as they are superposed. On the other hand for small angles, 2α, between these waves the nonlinear theory indicates formation of a 'wave jump'. Such jumps were identified by Yue and Mei (1980) and further studied by Peregrine (1983). They give rise to a "Mach stem" type of interaction between the two wave fields. The height of the Mach-stem may be greater or less than twice the height of the incident waves as may be seen from careful study of figures 3 and 4 in Peregrine (1983). One figure is for deep water waves, the other for solitary waves ("Wedge angle" on these figures should be identified with α here).

In the above-mentioned figures amplifications as high as four times the incident waves appear possible. Such amplifications have not been observed in experiments and there are indications that diffraction effects may limit all or most of these excessive amplifications especially in deeper water. For solitary waves the maximum amplification observed by Melville (1980) in experiments on Mach stem reflection was no greater than twice, but Funakoshi's (1980, 1981) computations with the Boussinesq equations do include an example with an amplification of nearly three times.

Numerical solutions

The linear superposition of waves described above, gives an exact linear solution has been compared with solutions of a linear parabolic equation,

$$2ikA_y = A_{xx}$$

for wave amplitude $A(x,y)$ where surface elevation is $\eta = Ae^{iky-\omega t}$, k is the wavenumber and y is in the direction from the initial line to the focus. As expected good agreement was obtained up to $\alpha = 30°$ the maximum value considered.

Nonlinear solutions were found by numerically integrating the nonlinear Schrödinger equation

$$2ikA_y = A_{xx} - K|A|^2A,$$

as described by Yue and Mei (1980). Some computation was with a simple implicit finite difference scheme, but for most computations a substantially more efficient high-order explicit scheme due to Dold is used. Integration time on a multi-user VAX computer is of the order one minute elapsed time, so results are readily available for any choice of parameters. Figure 2 shows an example corresponding to the case in figure 1(b) depicting the wide focussing region and bordering wave jumps with modulations as described by Peregrine (1983). A useful way of interpreting this type of example is that the focussing part of the wave contributes an extra width to the Mach-stem regions the bounding plane waves would create on their own. However, there are further aspects of this area which are to be described in a more complete account in preparation. For comparison figure 3 shows a linear solution for the same initial conditions.

Experimental measurements

Experiments corresponding to the examples described above were carried out in the Wide Wave Tank at Edinburgh University. This tank has 75 wave paddles each one foot (30cm) wide. A line of 24 wave gauges were used to measure wave height. Any chosen wave pattern was repeated 16 times, on each occasion being moved along by one paddle. In this way a set of 24 × 16 wave measurements were made without changing equipment position. Under computer control this gave a whole range of experimental results at different steepnesses, and focussing distances with both the wave patterns indicated in figure 1.

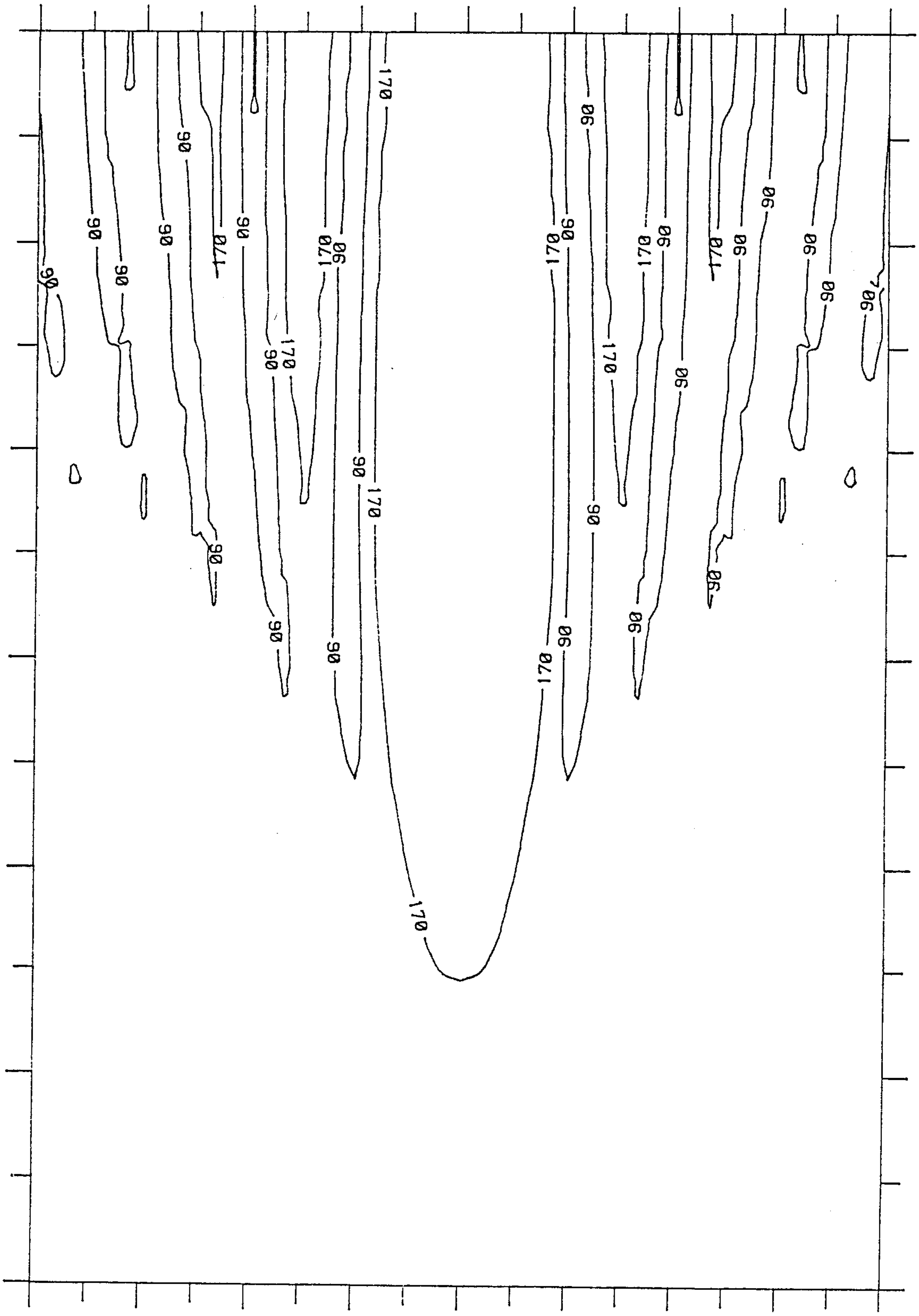

Figure 2 A contour plot of a numerical solution for the amplitude of a wave, like that in figure 1(b), with uniform initial steepness ak = 0.2, H/L = 0.06. This shows the very wide focussing region and the modulated wave jumps at each side. Contours are at 10%, 90% and 170% of the initial amplitude.

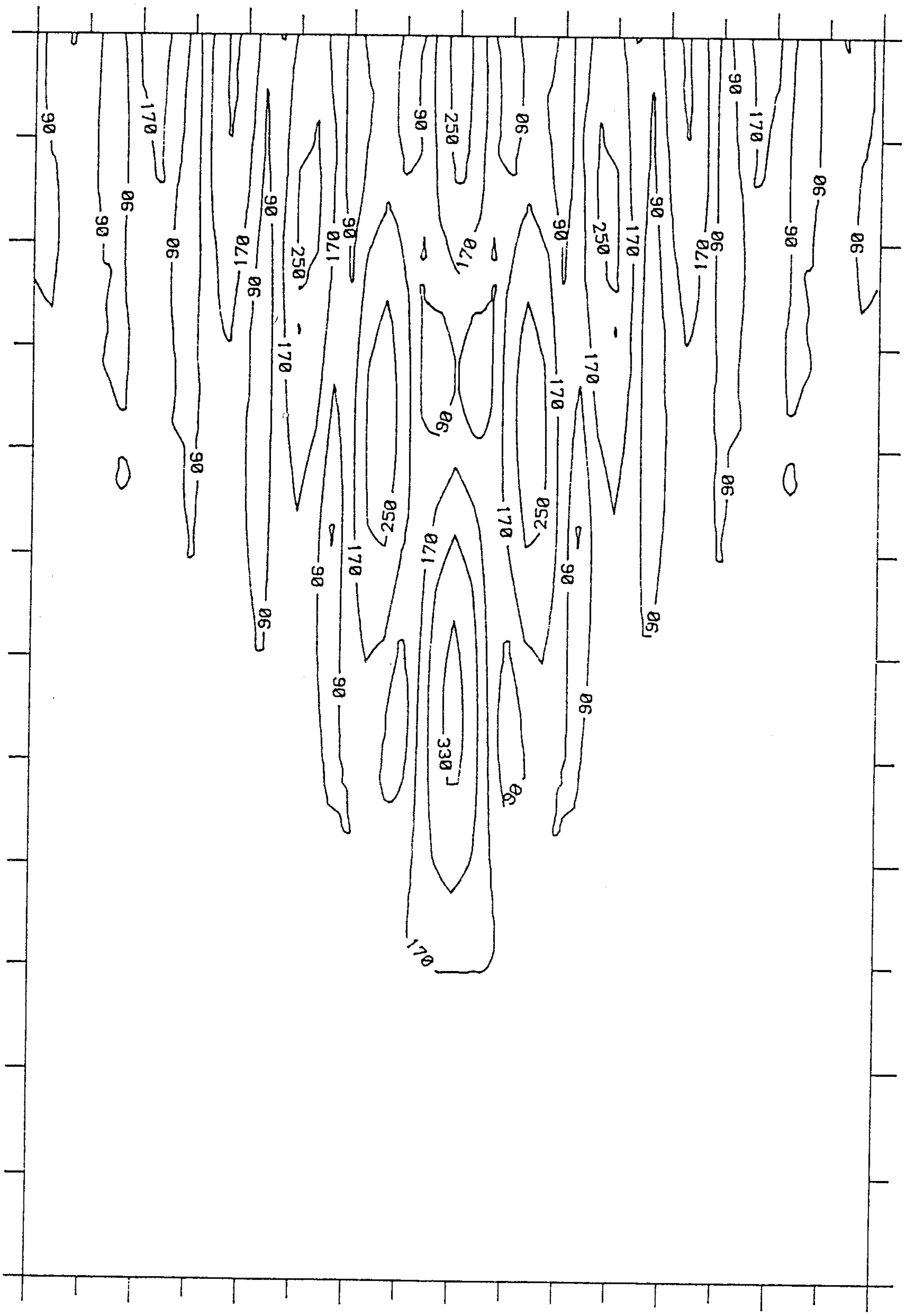

Figure 3 The linear theory solution equivalent to figure 2. Contour intervals are the same, but also include 250% and 330% of the initial amplitude, which are not reached in figure 2

In figure 4(a) we show, experimentally measured wave amplitudes for example F. The amplitude of the measured Fourier component at the forcing frequency is shown. This is compared in figure 4(b) with the corresponding linear solution, and the amplitude is clearly much greater than in the experiment. The nonlinear solution in figure 4(c) is much closer to the experimental values. However, there are small scale features in the experiment which suggest the spatial modulation one may expect from a reflected waves. The numerical integration was continued "beyond" the end of the tank where it was reduced by a reflection coefficient corresponding to that measured for plane waves, 0.05 in this case, and the resulting reflected wave added to the wave field of figure 4(c) with the result shown in figure 4(d). A superposition of figures 4(a) and (d) given in figure 5 shows that although there are some differences agreement is satisfactory in many details.

It would be surprising if there were no discrepancy since the nonlinear Schrödinger equation describes weakly nonlinear waves and in this particular experiment the maximum wave steepness is 85% of the steepness of the steepest steadily propagating wave.

Conclusions

Experiment and theory for wave focussing are in good quantitative agreement once reflection from the end of the tank is allowed for. In some experiments waves just reached breaking steepness.

Nonlinear defocussing of water waves is confirmed, but the importance of linear diffraction in defocussing has also been noted and this can be dominant in practical cases.

The experimental tank length is insufficient for wave jumps to be studied but the results give confidence in the equations describing their existence, even for waves close to breaking.

A definitive report of this work is in preparation. The support of the U.K. Science and Engineering Research Council is gratefully acknowledged, and we thank Professor S. Salter for the use of the Wide Wave Tank at Edinburgh University.

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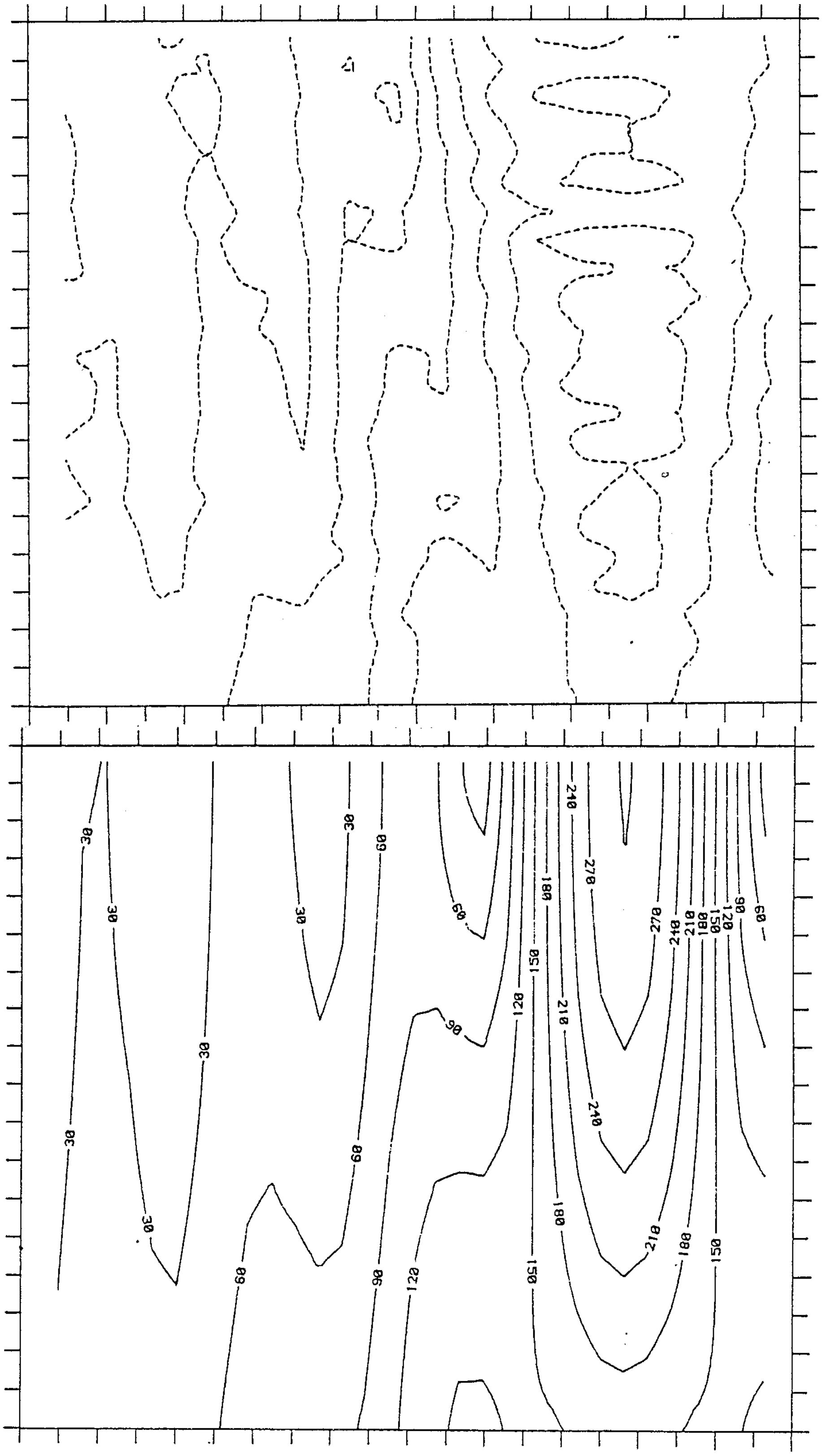
(a)

(b)



<u>Figure 4</u> Contour plots of wave amplitude in the region of experimental measurements for the wave pattern of example F with initial maximum steepness of ak = 0.2, H/L = 0.06. Contours at intervals of 30% of initial amplitude. (a) experiment (b) linear theory.

(c)

(d)

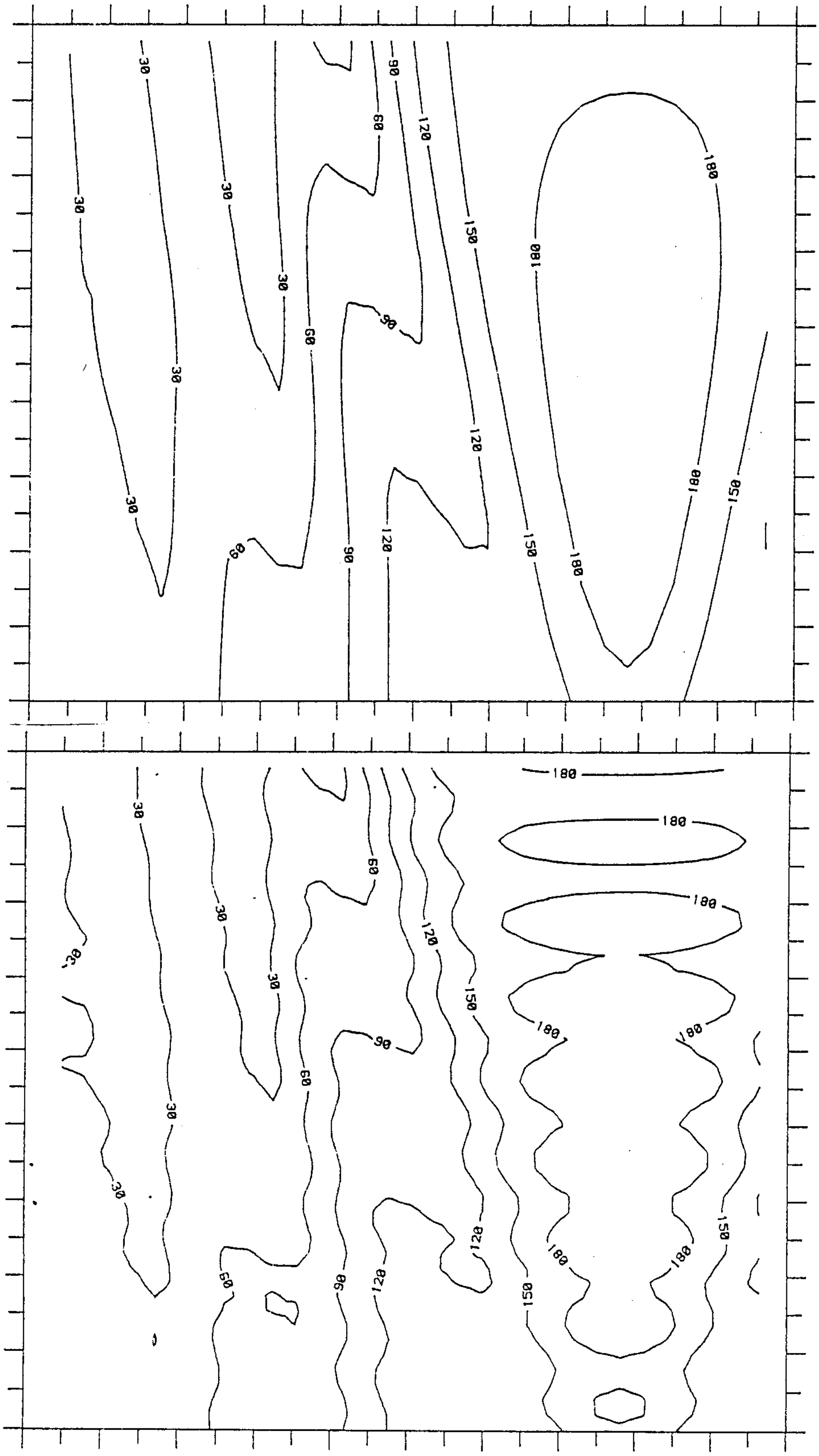


Figure 4 Contour plots of wave amplitude in the region of experimental measurements for the wave pattern of example F with initial maximum steepness of ak = 0.2, H/L = 0.06. Contours at intervals of 30% of initial amplitude. (c) nonlinear theory (d) nonlinear theory plus 5% reflection.

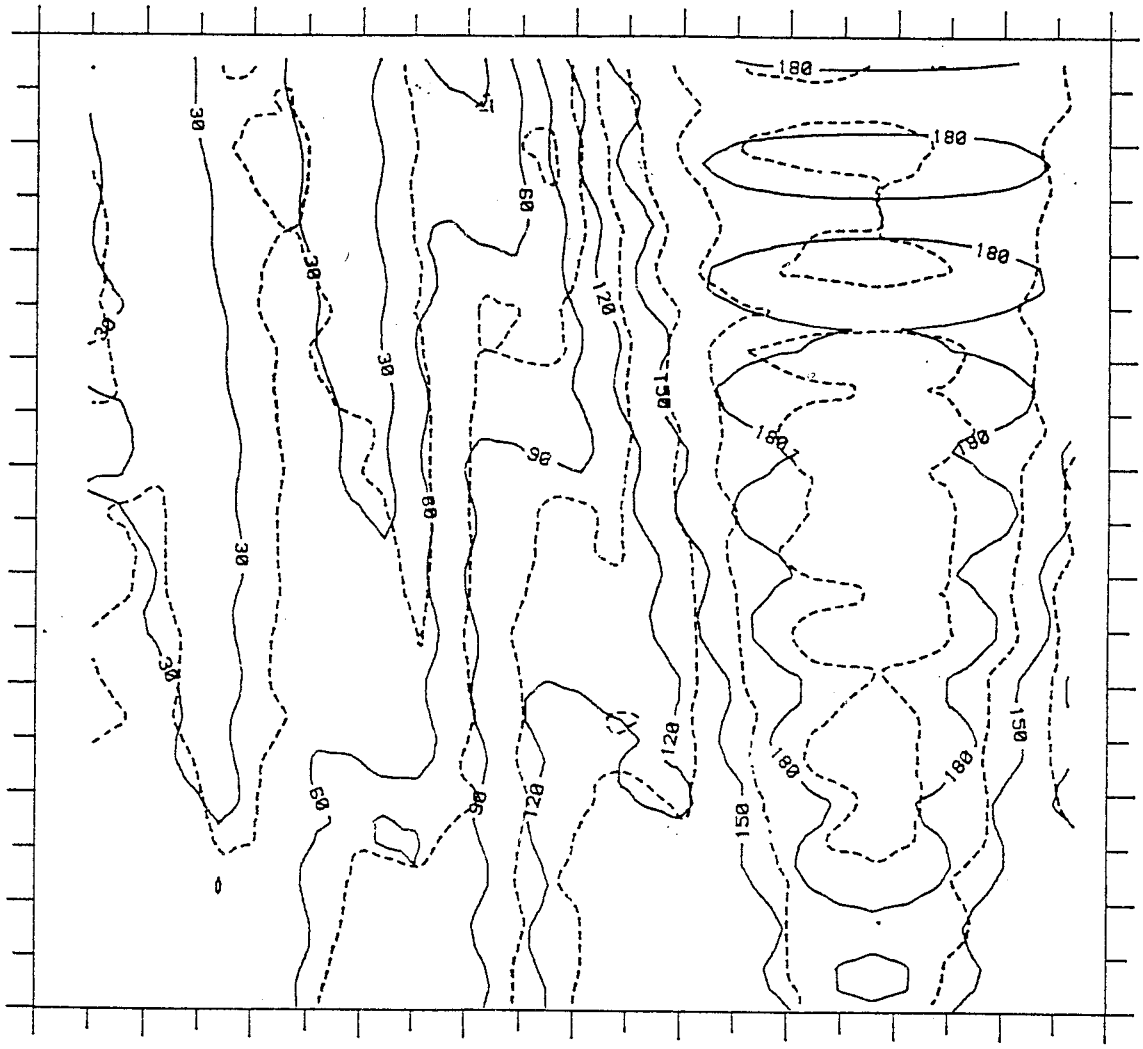


Figure 5 Figures 4(a) and 4(d) superimposed.
The broken lines are the experimental contours.