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SCATTERING OF WATER WAVES BY A SYSTEM OF VERTICALLY FLOATING PLATES

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Abstract

The two dimensional linear problem of wave radiation and diffraction by a vertically floating plate in water of infinite depth was solved analytically by Haskind [1]. In the present study we utilize this solution in order to study the behaviour of systems composed of several plates. Two types of systems are considered:

(i) systems of rigidly held plates; and (ii) so-called free systems, in which the plates are only connected to each other. An approximation of wide spacing between the plates is assumed throughout the formulation of the governing equations. For each plate the far field solution is matched with those of the adjacent plates. This makes it possible to present the solution in a closed, relatively simple form. Results, including the transmission and reflection coefficients as well as the displacement amplitudes of the structures for various systems are presented and discussed. Wave-flume experiments verify the validity of the theoretical approach, but also show the importance of viscous dissipation. The two main conclusions regarding the possible operation of such structures as floating breakwaters are as follows. First, for rigidly held systems, in contrast with what one might expect, the transmission coefficient does not generally decrease when increasing the number of plates. Second, and probably more important, increasing the number of plates in the case of freely floating systems reduce their overall motion, render their behaviour more similar to that of fixed systems, and causes a decrease in the transmission coefficient.

1. Introduction

The advancements made in recent years in the field of offshore engineering have led to an increased interest in floating breakwaters. A major purpose of such a device would be to offer protection against incoming surface waves in deep water where conventional breakwaters are impractical. In what follows we present mathematical and experimental studies of a few systems, leading to what seems to be a prospective solution. The systems proposed herein are composed of several vertically floating plates, parallel to each other, and interconnected through hinges by means of horizontal rigid bars, see Fig. 1.

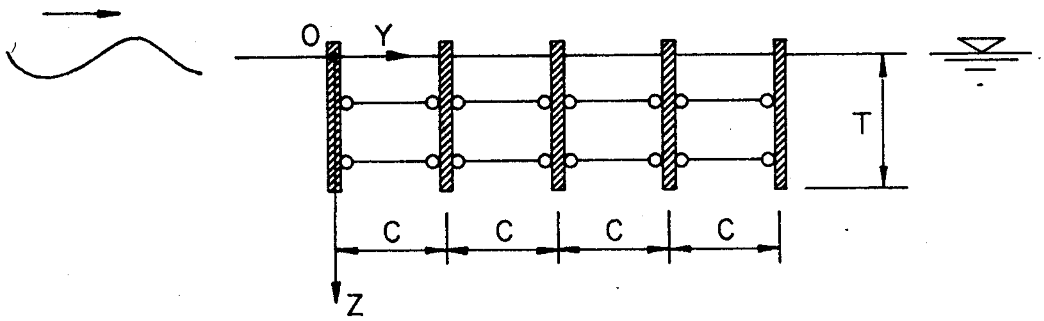

Fig. 1: A sketch of the system

We consider a two dimenstional model in the (y,z) plane. Physically this is represented by infinitely long plates perpendicular to this plane. The y axis coincides with the undisturbed water surface and the z axis is pointed downwards. We assume an irrotational flow in infinitely deep water and remain within the framework of linear wave theory. Thus, the governing equation and free-surface boundary condition are

$$\phi,_{yy} + \phi,_{zz} = 0 \quad , \quad z \geq 0 \tag{1.1}$$

$$\phi,_{\tau\tau} - g\phi,_{z} = 0 \quad , \quad z = 0 \tag{1.2}$$

in which ϕ is the velocity potential, g is the gravitational acceleration, τ is the time, and where subscripts preceded by a comma denote partial differentiation.

The potential of an incident wave, with frequency σ and unit amplitude, approaching the system from the left ($y = -\infty$) is given by

$$\phi_0 = -(jg/\sigma)\exp[j\sigma t - k(z+jy)], \qquad k = \sigma^2/g \quad , \quad j = \sqrt{-1} \tag{1.3}$$

The complete formulation of the mathematical problem is given by eqs. (1.1), (1.2), (1.3), proper boundary conditions on the plates, and the so-called radiation condition at infinity.

In section 2 we summarize the closed mathematical solution for a single plate as given by Haskind [1] and Stiassnie [4]. An experimental verification of this solution is also presented in this section. In section 3 we utilize the wide spacing assumption (Srokosz and Evans [3]) to obtain a formulation in the form of a linear algebraic system of equation for the multiplate configuration. The computed results for multiplate systems are presented and discussed in section 4. Comparison to some experimental measurements in a wave flume is included in the same section. Details about the experimental setup are given in Appendix A.

2. Single Plate

The case of a single plate was studied extensively by quite a few investigators. The following is based on the results of Hasking [1] and Stiassnie [4]. The transmission coefficient - Tc (defined as the transmitted-to-incident wave amplitude ratio) for a thin plate, submerged to a depth T, is given by:

$$Tc = t + B_2H + B_4A \qquad (2.1)$$

Where t is the transmission coefficient for a rigidly held plate, H is the ratio between the horizontal displacement amplitude of the point O (intersection between the plate and the undisturbed water surface) and the incident wave amplitude; A is the amplitude of the angular motion about O per unit incident wave amplitue. Thus $\tilde{H} = H + A \cdot T$ is the ratio between the horizontal displacement amplitude of the lower edge and the incident wave amplitude. For the case of a weightless freely floating plate the expressions for H and A are:

$$H = (-Y_g D_{44} + M_g D_{24})/D \quad ; \quad A = (Y_g D_{42} - M_g D_{22})/D \qquad (2.2)$$

The physical meaning of the various quantities appearing in eqs. (2.1) and (2.2) is as follows: B_2, B_4 are the amplitudes of the waves radiated in the positive direction by a unit amplitude of horizontal and angular displacements, respectively, for a single plate. Y_g, M_g are the force and moment exerted on the plate by a unit amplitude wave arriving from the left. We also have

$$D_{pq} = \sigma^2\mu_{pq} - j\sigma\lambda_{pq} \quad , \quad (p \text{ and } q = 2,4) \; ; \; D = D_{22}D_{44} - D_{24}D_{42} \qquad (2.3)$$

where μ_{pq} are the added mass coefficients and λ_{pq} are the damping coefficents. The index 2 reffers to horizontal motion and 4 to angular motion. The detailed mathematical expressions for the above mentioned quantities are rather long and are given in Appendix B.

For a rigidly held plate the amplitudes H and A vanish so that Tc = t. For this case the variation of |Tc| and the reflection coefficient |Rc| are shown in Fig. 2 as a function of T/λ (λ being the wave length given by 2π/k). In Fig. 2, as well as in other figures, the general notation is such that the theoretical solutions are represented by continuous curves (full or dashed lines), and the experimental results are denoted by discrete data points.

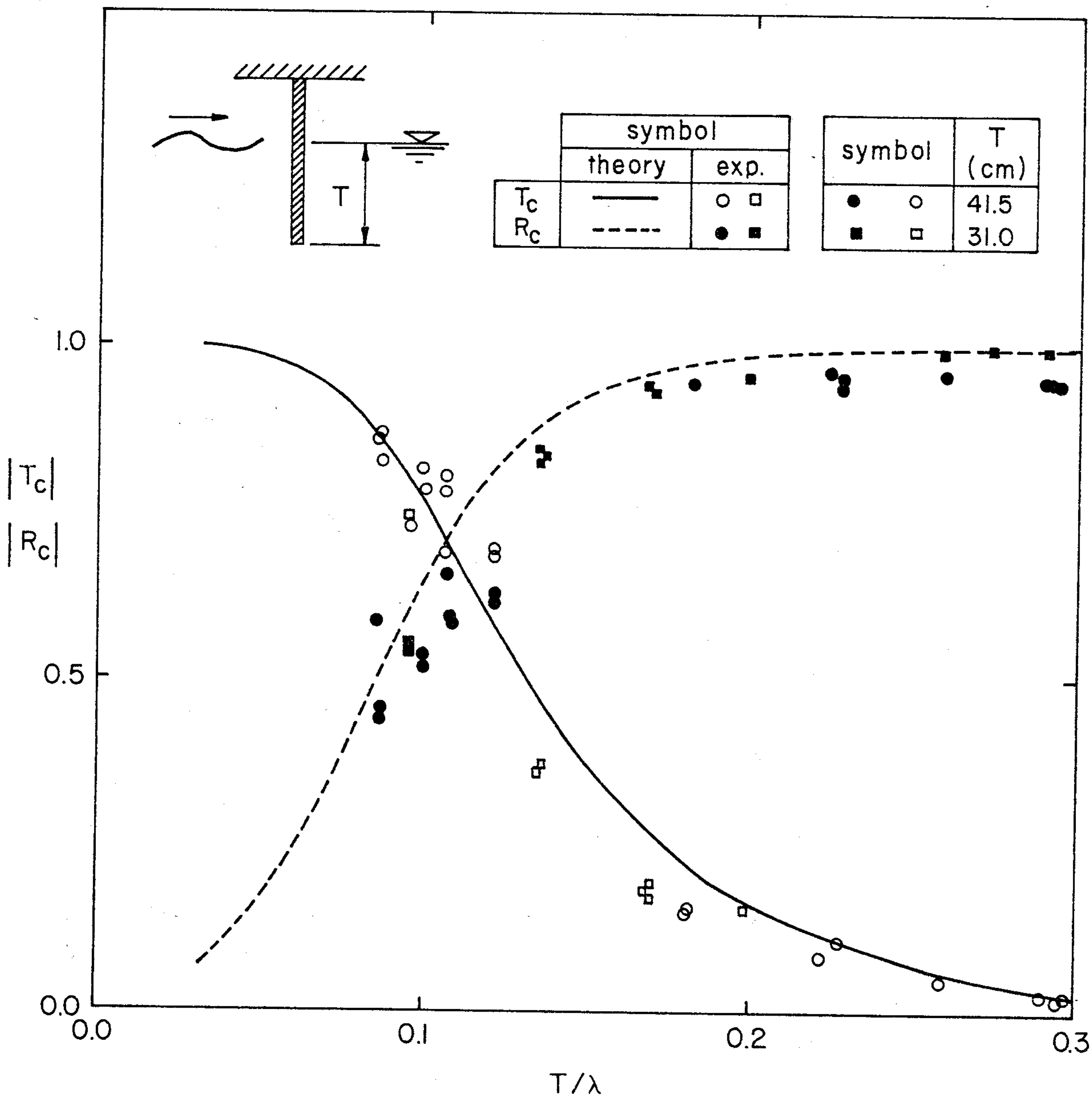

Fig. 2. The transmission and reflection coefficients for a rigidly held single plate.

The agreement between the theoretical and experimental results for this simple case is satisfactory.

In order to assess the validity of the computational model for more complicated cases we conducted a series of experiments with a floating plate held by means of horizontal linear springs. The solution for this case was presented by Stiassnie [4]. The springs were attached to the plate at the point z = 0 (i.e., at the free surface). This was an arbitrary choice, but it bears no loss whatsoever on the generality of the solution. A detailed description of the experimental setup is given in Appendix A. An example of the results for this case is shown in Fig. 3.

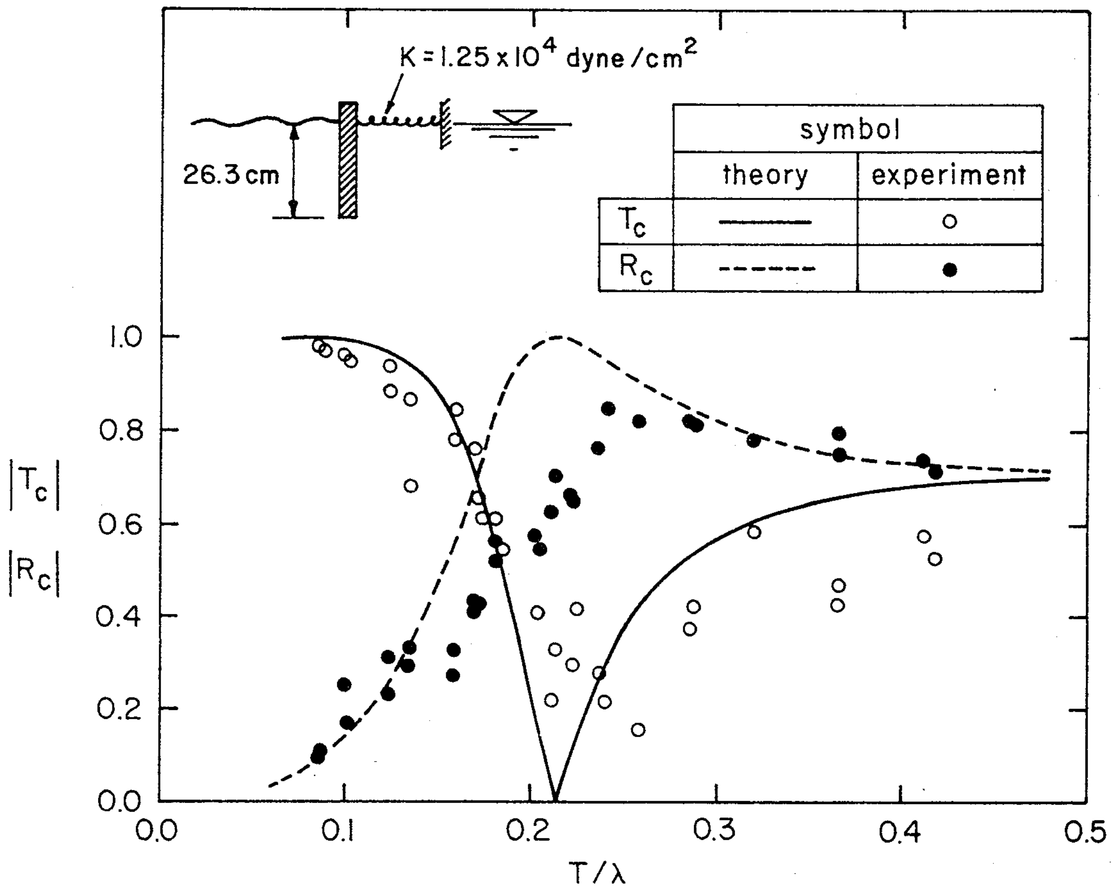

Fig. 3. The transmission and reflection coefficients for a spring-moored single plate.

The agreement between the theoretical and experimental results for the spring-moored plate appears to be fair. It is certainly not as good as that for the fixed plate. The reason for that is probably due to more energy dissipation which is enhanced by the motion of the plate.

The rather long, complicated mathematical expressions may give rise to computational mistakes. One simple check for such mistakes is the energy balance which should yield $|Rc|^2 + |Tc|^2 = 1$. The computed values of Tc and Rc were found to comply with this requirement.

The generally favorable agreement between the computations and the experiments, as well as the energy balance check of the computational results confirmed the validity of the single plate mathematical model. In the next section we use this model as the basic element for the construction of the mathematical solution for systems composed of several floating plates.

3. Method of Solution for Multiplate Systems

Two types of multiplate systems are considered, (i) systems of rigidly held plates, and (ii) so called free systems, in which the plates float freely and are only connected to each other*.

Out of several existing biplate models, the one developed by Srokosz and Evans [3] which is based on the wide spacing assumption, seems most appropriate to be generalized and applied to the computation of a multiplate system. The assumption of wide spacing means that the plates are spaced far enough from one another, so that the local wave field in the vicinity of one plate does not influence the other plates.

The only interaction between the plates is due to the far field propagating wave terms which appear in the radiation and scattering problem for a single plate. Mathematically this is equivallent to requiring the wave length λ to be small compared to the distance c between any two adjacent plates, but there is an accumulating evidence in the literature that the method is valid over a much wider range of λ/c values.

Let us observe now N identical and evenly spaced plates as shown in Fig. 4. The terms R_i, i = 1,2,...,N, denote the amplitudes of the waves travelling to the right. The amplitudes of the waves travelling to the left are denoted by L_i, i = 1,2...N . The subscript i indicates that the wave approaches the i-th plate (either from its right,or left,side). The numbering of the plates is sequential from left to right. Without loss of generality the incident wave from the left was considered to have a unit amplitude, i.e., $R_1 = 1$. L_N was set equal to zero in accordance with the radiation condition.

* Despite being termed "free", in practice, these systems should be anchores against the non-oscillatory second-order drift forces.

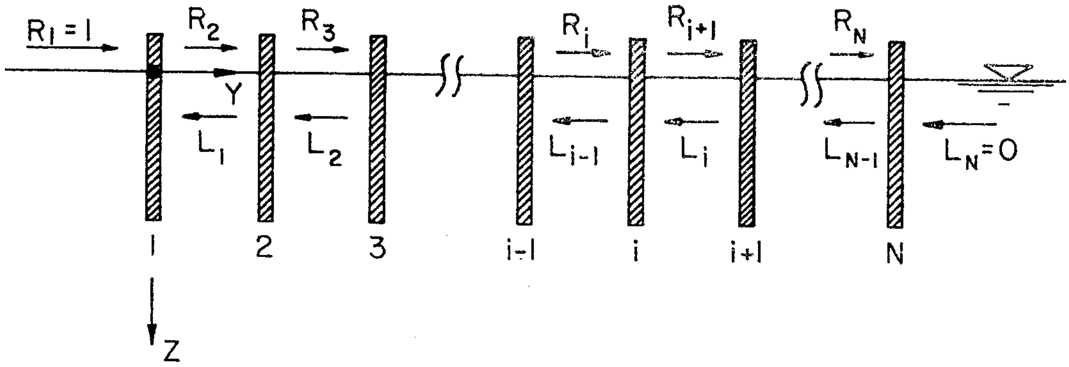

Fig. 4. Definition sketch of unknowns

In the case of free systems each two adjacent plates are connected by a pair of rigid bars (through hinges), thus granting the whole system only two degrees of freedom. All plates must have the same horizontal - (H) and angular (A) movements.

H, A, R_i (i = 2,...,N) and L_i(i = 1,...,N-1), constitute the 2N unknown variables of the problem.

The equations of motion for the horizontal and angular movements are

$$N \cdot D_{22} \cdot H + N \cdot D_{24} \cdot A + Y_g \sum_{i=1}^{N} (R_i - L_i) = 0 \qquad (3.1)$$

$$N \cdot D_{24} \cdot H + N \cdot D_{44} \cdot A + M_g \cdot \sum_{i=1}^{N} (R_i - L_i) = 0 \qquad (3.2)$$

An implication of the wide spacing assumption is that the far field to the right of the i-th plate in identical with the far field to the left of the i+1 plate, for i=1,...,N-1. In this way we obtain a pair of equations for every interval between the plates: one for the wave propagating in the positive direction and the other for the wave propagating in the negative direction.

The N-1 equations of the first type and the N-1 equations of the second type, respectively, are:

$$B_2 \cdot H + B_4 \cdot A + t \cdot R_i + r \cdot L_i - E \cdot R_{i+1} = 0 \quad , \ i = 1,\ldots,N-1 \qquad (3.3)$$

$$-B_2 \cdot H - B_4 \cdot A - E \cdot L_i + r \cdot R_{i+1} + t \cdot L_{i+1} = 0 \quad , \ i=1,\ldots,N-1 \qquad (3.4)$$

in which r is the reflection coefficient for a rigidly held single plate and where E = exp (jkc). Equations (3.1) to (3.4) consistitute together a linear algebraic system of 2N equations with the same number of unknowns. For a fixed system H and A are identically zero and the relevant set of equations is given by a reduced form of (3.3),(3.4) which leads to a symmetric 3-diagonal matrix.

For two plates the solution is quite simple, but for large values of N the computation becomes tedious. In the following we present a method which makes it possible to perform simple computations for some very large values of N. The transmission coefficent and the displacement amplitudes for double-body systems are given by:

$$Tc = (E+r+t) \cdot R_2 + E \cdot t/(r-E) \qquad (3.5)$$

$$H = 2 \cdot (D_{44} \cdot Y_g - D_{24} \cdot M_g) \cdot [(r+t-E)/(E-r) - 2R_2]/D \qquad (3.6)$$

$$A = 2 \cdot (-D_{42} \cdot Y_g + D_{22} \cdot M_g) \cdot [(r+t-E)/(E-r) - 2R_2]/D \qquad (3.7)$$

where

$$R = \frac{-E \cdot t \cdot D + (E-r-t) \cdot YM}{(r-E)[(r+E) \cdot D + 2 \cdot YM]} \ ;$$

$$YM = 2 \cdot Y_g \cdot (D_{44} \cdot B_2 - D_{42} \cdot B_4) + 2 \cdot M_g(-D_{24} \cdot B_2 + D_{22} \cdot B_4) \qquad (3.8)$$

It will now be shown how the solution for a system with $N=2^{(n+1)}$, n = 1,2,... plates, can be calculated easily from the coefficients of a subsystem composed of 2^n, i.e., half the number of plates. We simply consider the system as being composed of two subsystems, two-bodies. To obtain the solution for the original system (i.e., with $N=2^{(n+1)}$) we have to substitue coefficients, appropriate to these bodies (each with $N=2^n$), into eqs. (3.5) to (3.8). The coefficients for there bodies are denoted by the superscript (n) and are calculated by the following recursive expressions:

$$r^{(n)} = \frac{r^{(n-1)}\cdot(t^{(n-1)})^2}{E^2 - (r^{(n-1)})^2} + r^{(n-1)} \;; \qquad t^{(n)} = \frac{E\cdot(t^{(n-1)})^2}{E^2 - (r^{(n-1)})^2}$$

$$B_2^{(n)} = B_2^{(n-1)}\cdot\left(1 + \frac{t^{(n-1)}}{E + r^{(n-1)}}\right) \;; \qquad B_4^{(n)} = B_4^{(n-1)}\cdot\left(1 + \frac{t^{(n-1)}}{E + r^{(n-1)}}\right)$$

$$Y_g^{(n)} = Y_g^{(n-1)}\cdot\left(1 + \frac{t^{(n-1)}}{E + r^{(n-1)}}\right) \;; \qquad M_g^{(n)} = M_g^{(n-1)}\cdot\left(1 + \frac{t^{(n-1)}}{E + r^{(n-1)}}\right)$$

$$D_{22}^{(n)} = 2\cdot\left(D_{22}^{(n-1)} + \frac{B_2^{(n-1)}\cdot Y_g^{(n-1)}}{E + r^{(n-1)}}\right) \;; \qquad D_{44}^{(n)} = 2\cdot\left(D_{44}^{(n-1)} + \frac{B_4^{(n-1)}\cdot M_g^{(n-1)}}{E + r^{(n-1)}}\right)$$

$$D_{24}^{(n)} = 2\cdot\left(D_{24}^{(n-1)} + \frac{B_4^{(n-1)}\cdot Y_g^{(n-1)}}{E + r^{(n-1)}}\right) = D_{42}^{(n)} = 2\cdot\left(D_{42}^{(n-1)} + \frac{B_2^{(n-1)}\cdot M_g^{(n-1)}}{E + r^{(n-1)}}\right) \qquad (3.9)$$

Note that quantities with superscript zero are those for a single plate and are given explicitly in Appendix B.

4. Results and Discussion

Before going into details we would like to emphasize that our main effort, in the present research was aimed at analysing the performance of floating structures as breakwaters. Thus, most of the results are presented from a "breakwater oriented" point of view. For the sake of this discussion we have arbitrarily chosen the criterion $|Tc| \leq 0.2$ as a required condition for a structure to be considered as an efficient breakwater. To meet this criterion by means of a single plate would require $T/\lambda \gtrsim 0.2$ in the case of a rigidly held plate and $T/\lambda \gtrsim 2$ for the freely floating one. Both of these requirement seem difficult to meet. For water of very great depth it appears impractical to consider rigidly held breakwaters. Just as well, and quite obviously, floating structures with a draft of about twice the commonly observed wave lengths are also impractical. An alternative structure, namely a flexibly moored plate, was discussed by Stiassnie [4], but this solution was also found to be infeasible, as it is associated with huge ascillatory forces which have to be transferred to the ground far below.

The two basic assumptions that motivated the multiplate research were:
(i) A fixed system increases its efficiency with the increase of the number of plates; and (ii) The performance of a free system gets more similar to that of

a fixed one with increasing the number of plates. In the following it is shown that we were wrong in the first assumption but were, fortunately, right in the second one. The variations of the transmission coefficient for multiplate systems are shown in Fig. 5 as a function of the number of the plates and the spacing to wave-length ratio*. In this figure, as well as other figures, the shaded zones are those with $|Tc|$ values greater than 0.2 and white zones are those with values smaller than 0.2. Figures 5a and 5b are respectively, for fixed and free systems with $T/\lambda = 0.1$. Figures 5c and 5d are for fixed and free systems with $T/\lambda = 0.2$. Inspection of these figures leads to the following conclusions: (i) the performance of the fixed system, in contrast with what one might expect, is practically the same for almost any large value of N; (ii) The advantage of systems with $T/\lambda = 0.2$ compared to those with $T/\lambda = 0.1$ is substantial; and (iii) for N greater than about 16, free systems start to behave similarly to fixed ones. To strengthen these conclusions we show, side by side, the transmission coefficent as a function of c/T and T/λ for two fixed plates (Fig.6a) and for a free systems composed of sixteen plates (Fig. 6b). There exists a clear resemblance between these two figures. It seems that the narrow hyperbolically shaped zones with $|Tc| > 0.2$ are related to the possible creation of standing waves between the plates. This happens whenever $c \cong n(\lambda/2)$; $n = 1,2,...$. The fact that a multiplate free system behaves similarly to a fixed one is probably due to the accumulating added masses which sum up to a large added mass value. The horizontal displacements at the water surface ($|H|$) and at the bottom of the plates ($|\tilde{H}|$) for a system of 16 plates are shown in Figs. 7a and 7b respectively. These figures clearly indicate the significantly reduced movements of the 16-plate structure, which again supports the observation of resemblance between the performance of the free and fixed multiplate structures.

The results presented above were computed for systems of constant drafts and with even spacings between the plates. Further computations for uneven spacings and variable drafts showed some minor variations from the above results. They certainly did not yield any considerable improvement, as far as the practical purpose of the device is concerned.

The experimental study with multiplate systems was limited, at least for the time being, to fixed structures. Furture studies will probably include the more complicated experimental setups of free multiplate structures.

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*Note that the mathematical solution is periodic in c/λ, with period one.

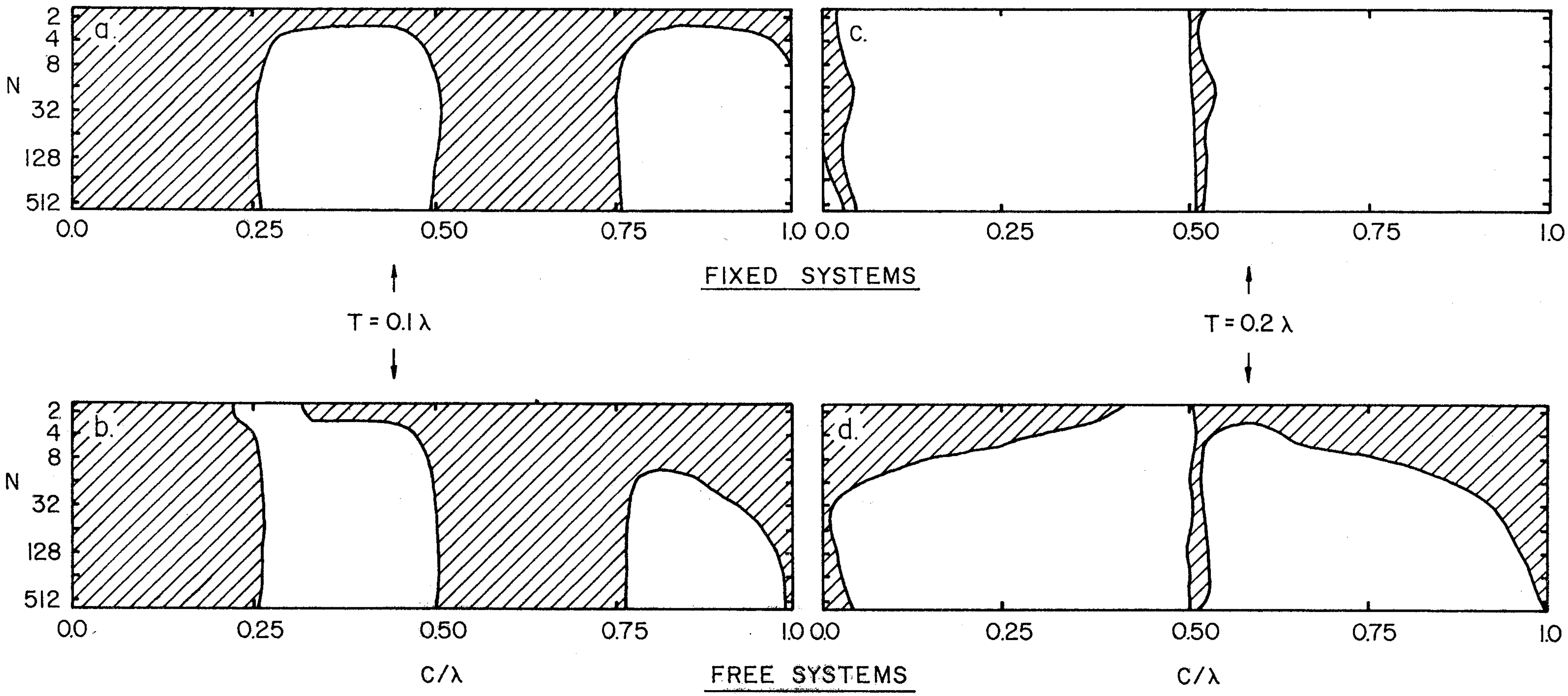

Fig. 5. The transmission coefficient isolines for $|Tc| = 0.2$. a. fixed systems with $T/\lambda = 0.1$; b. free systems with $T/\lambda = 0.1$; c. fixed systems with $T/\lambda = 0.2$; d. free systems with $T/\lambda = 0.2$.

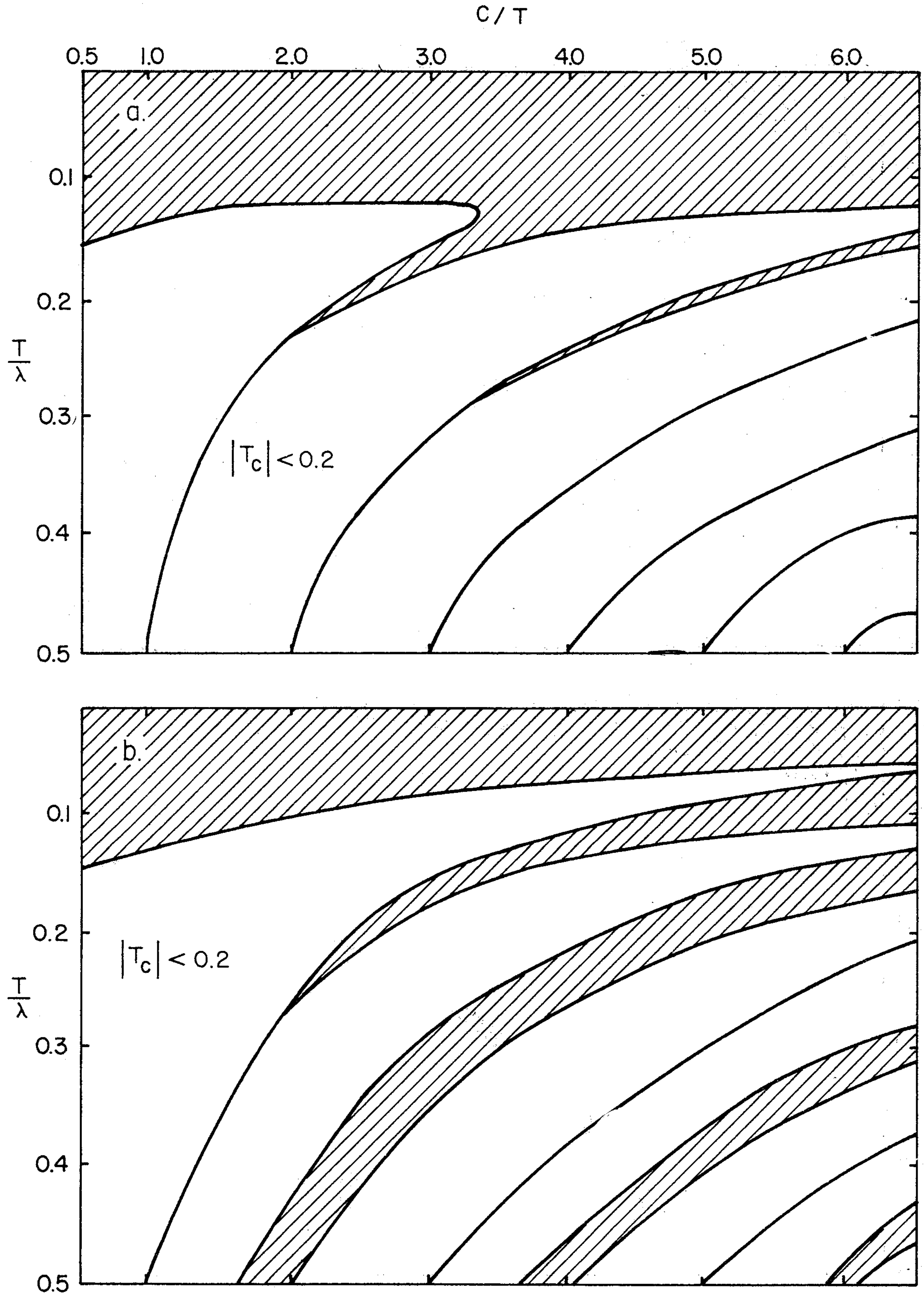

Fig. 6. The transmission coefficient isolines for |Tc| = 0.2;
a. for two fixed plates; b. for 16 freely floating plates.

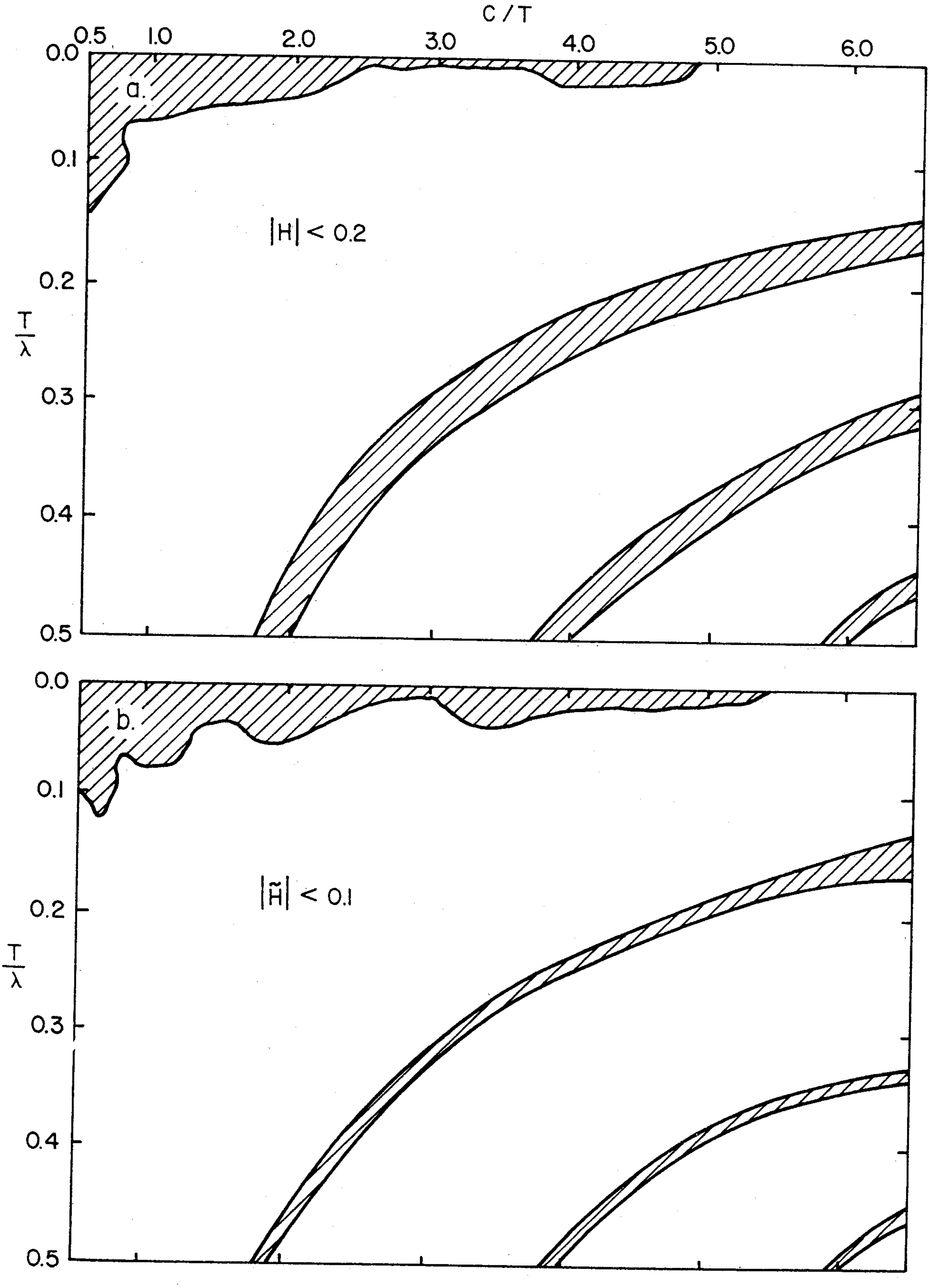

Fig. 7. Isolines of the horizontal displacement amplitudes.
a. at the water surface ($|H| = 0.2$); b. at the lower edge ($|\tilde{H}| = 0.1$).

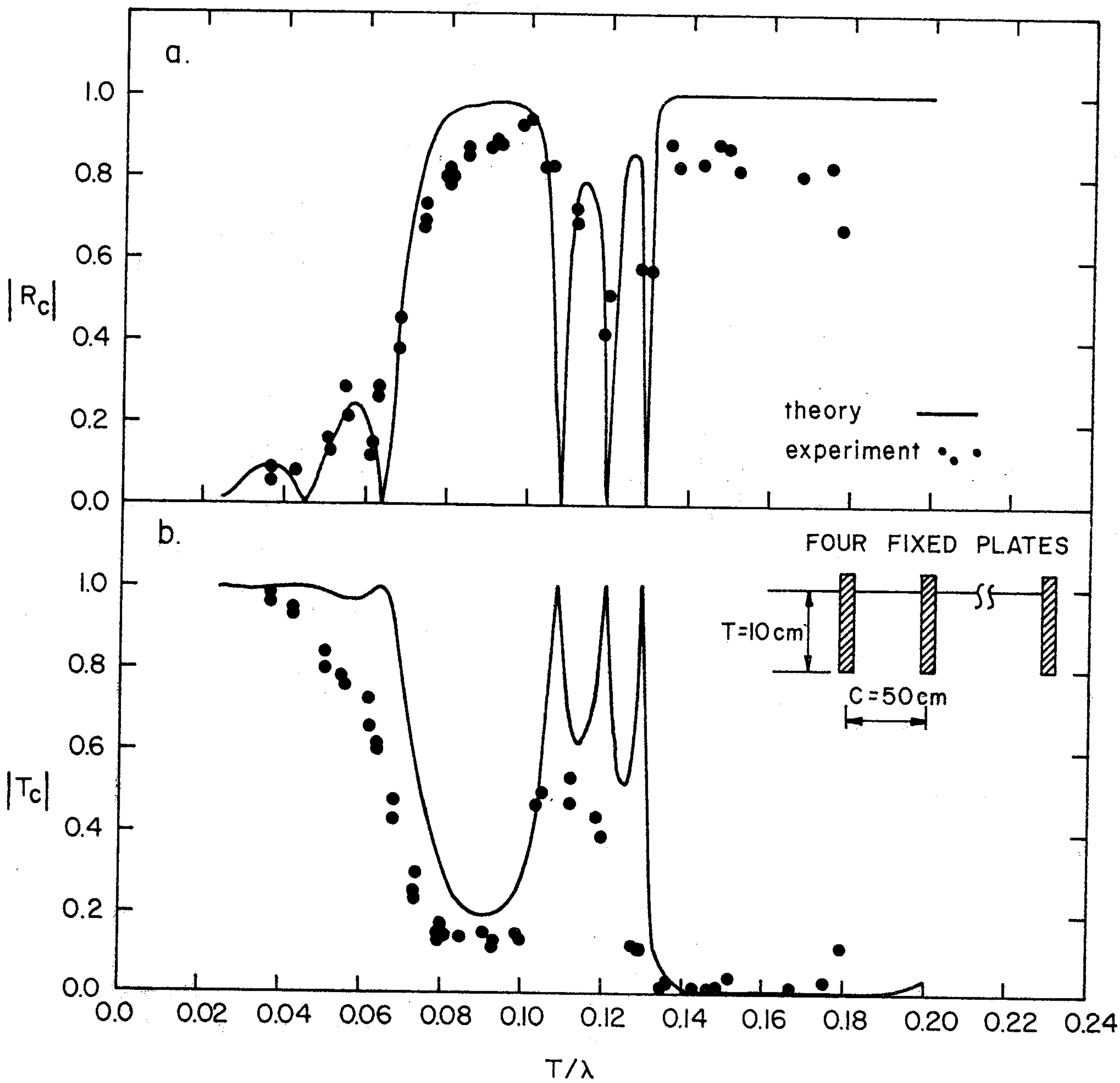

Fig. 8. Four-plate fixed system. a. Reflection coefficient;
b. Transmission coefficent.

The results for fixed systems of four and eight plates are shown in Figs. 8 and 9 respectively.

Inspection of the results from all of the experimentally tested fixed systems (Figs. 2, 8 and 9) indicates the significance of energy dissipation as follows. First, it appears that the agreement between the (no-dissipation) theory and experiments is better for the smaller number of plates. The energy dissipation is caused mainly by the vortex shedding at the bottom edge of the plates, thus, systems with greater numbers of plates have more energy dissipators which are responsible for the poorer agreement between the theory and the experiments.

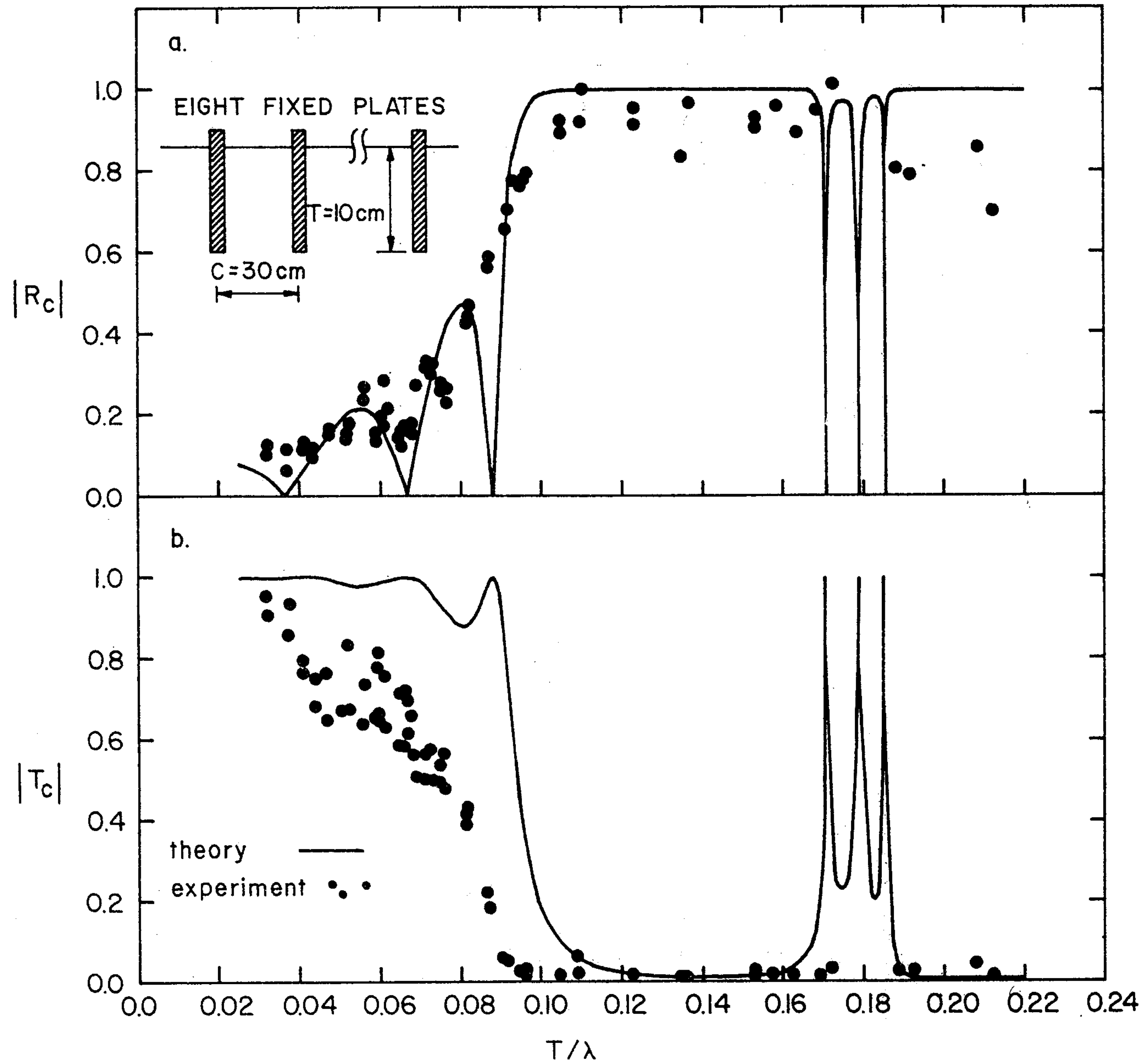

Fig. 9. Eight-plate fixed system. a. Reflection coefficient;
b. Transmission coefficient.

Second, the agreement, as seen in Figs 8 and 9, is better for the reflection coefficient than for the transmission coefficient. Intuitively, the reflection mechanism is practically confined to the first plates in the structure, it involves less energy dissipation than the transmission of energy through the entire structure. Finally, the experimental scatter of data points which is seen in Fig. 9b for $T/\lambda < 0.8$ was noted to be not just a random experimental scatter. Rather, waves of smaller amplitudes were found to be systematically closer to the theoretical curve. This again, implies the significance of energy dissipation which is controlled by a nonlinear, amplitude - dependant mechanism.

The amount of dissipated energy was found, in some cases, to reach as high as 70 percent of the total incident wave energy. As far as the practical purpose of the device is concerned, this is a favorable aspect which has not been included in the present study. Hopefully, this aspect will be included in future studies, as it will certainly be helpful in the development of the actual structure.

5. Conclusions

The present study is a basic step in the development of one of the forms proposed for floating breakwaters. In view of its purpose, the main conclusions drawn from this study are:

(i) In contrast with what one might expect, in general, fixed systems with large number of plates show no improvement over systems with four plates.

(ii) Free systems improve their reforemance with increasing the number of plates up to between 16 and 32 plates. Beyond that the displacements of the system are very small, and it behaves essentially the same as a fixed one.

Note that conclusions (i) and (ii) are correct so far as energy dissipation is not included in the model.

(iii) The experimental model confirmes the validity of the mathematical approach.

(iv) Considering the practical purpose of the device, the experiments displayed the significance of introducing energy dissipation to the model.

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Appendix A: Experiments

A.1. The Wave Channel

The experiments were conducted in a 27m long, 60 cm wide, 1.30 m deep wave channel, which is equipped with a hydraulically powered piston-type wave generator. The model was installed approximately 16 m away from the wave generator, where two side windows of 6 m overall length enable to observe its behaviour during the experiments. A slope with rubberized hair at the downstream end of the channel was used as a wave absorber to minimize reflection. A porous screen of the same rubberized material was placed in front of the wave generator in order to filter out small disturbances and to absorb the waves which are reflected back and forth between the model and the plate of the generator.

A.2. The Generation of Waves

The wave generator is controlled (through a servo-valve) by an outside electronic signal. For the present study a sinusoidal function generator, with continuous frequency and amplitude scales, was used in order to produce monochromatic waves of desired frequencies and amplitudes.

A.3. The Wave Measurements

The waves were measured with resistance-type wave gages placed at fixed, accurately measured, locations along the wave channel. The signals from the gages were transmitted to an online minicomputer for immediate analysis. Four gages in front of the model were used to measure the incident and reflected waves. The transmitted waves were measured by four gages behind the model. The basic method in which the incident, reflected, and transmitted waves were determined is essentially the same as that employed by Naheer [2]. Actually, this method requires only two gages in front of the model and two gages behind it. However, a single measurement from two gages might result in errors which are difficult to estimate. Such errors may be due to wave nonlinearilies, small free waves overiaing the main wave, slight nonlinear responses of the measuring devices, and errors in measuring the distance between the wave gages. For a set of four wave gages there are six different combinations of pairs. Thus, instead of a single valued result (with an undetermined error) from one pair of gages, there are six values for which the average is taken as the representative result.

The method used to evaluate the waves from the records yields no solution when the distance between the wave gages is a whole multiple of half the wave length. In cases where this distance is close to that of no solution, small errors in distance measurement yield great errors in the results. These cases were excluded from the present calculations when the distance between the i-th and j-th wave gages was such that $-0.05 \leq \sin k (x_j - x_i) \leq 0.05$. In order to avoid cases in which all of the combinations of wave gage pairs fall in this category, the four gages were placed in the channel with unequal distances between them.

One of the basic theoretical assumptions is the linearity of the waves. However, linear wave records are hardly ever obtained in the laboratory, due to either wave nonlinearities, free waves, or nonlinear response of the wave gages. Even though the generated wave heights were very small (to insure linearity), it was impossible to always obtain linear records. Therefore the processing of experimental data included Fourier analysis of the records. The basic ("linear") modes of this analysis were used to calculate the incident, reflected and transmitted waves.

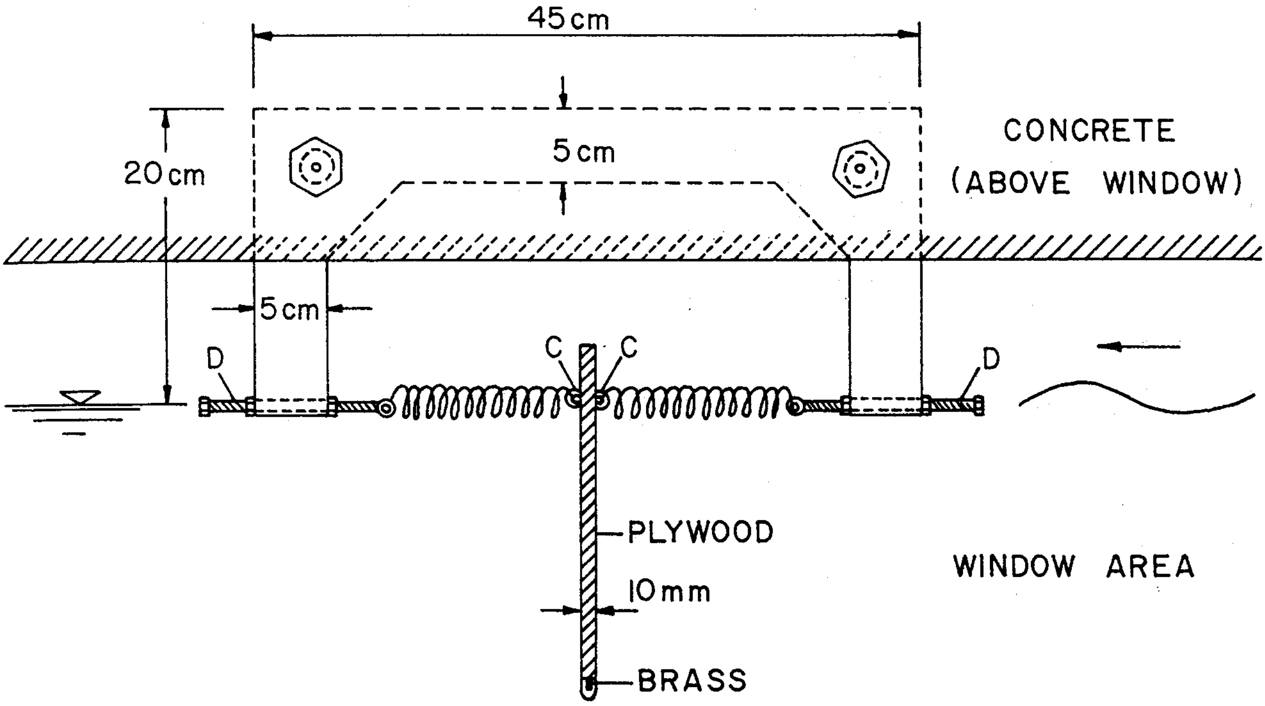
Cases of special interest were noted in some tests with the spring-moored single plate. In those tests the nonlinearities were surprisingly high, even with very small wave amplitudes. Spectral analysis of the wave records indicated that these nonlinearities were due to free waves of exactly twice the frequency of the incident wave. Review of the theoretical equations showed that the pressure at the bottom of the plate (which is the driving force of its heaving motion) oscillates at twice the frequency of the sway and roll motions. Indeed this kind of motion was observed in those experiments, and they were probably the cause for this particular sort of nonlinearity. However, further study is required to verify this quantitatively.

A.4. The Model

A.4.1. The Spring-Moored Single Plate

The plates were made of 10mm thick plywood with density of $0.53 gr/cm^3$. A 10mm thick brass strip with density of $8.47 gr/cm^3$ was attached to the side of the playwood in order for the plate to float vertically. The brass strips were machined to a prescribed width so that a given plate would float freely with a desired draft. Several plates were used with draft range from 20.0cm to 41.7cm. All the plates protruded approximately 4cm above the surface to prevent wave overtopping. This restricted the generation of wave heights to a maximum of approximately 4cm, which is unrelated to, but in compliance with the restriction imposed by the linearity requirement.

The plates were coated with special paint to prevent soaking and the consequent change of their mass and buoyancy.



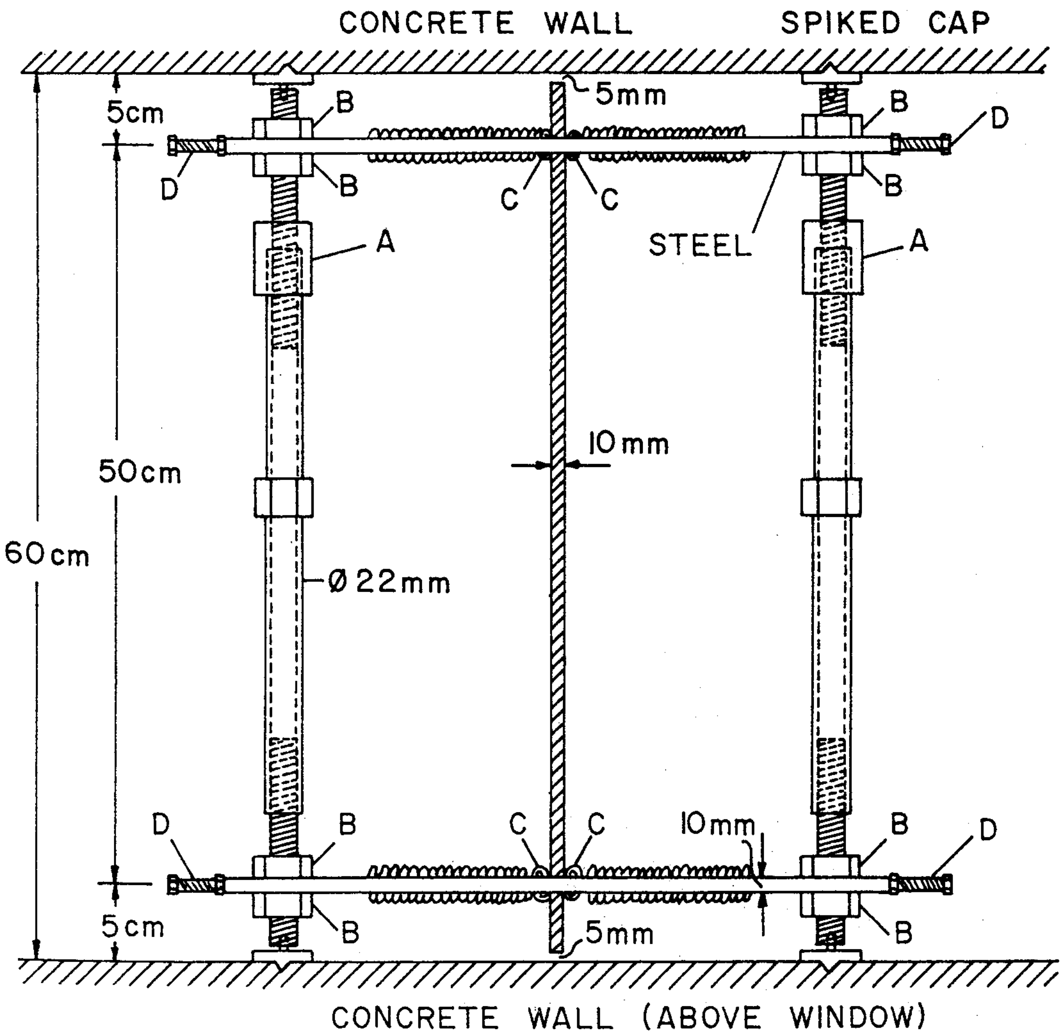


Fig.10. The experimental anchoring frame for the spring moored single plate.

The corners of the brass strip at the bottom of the plate were rounded to reduce energy losses due to flow separation at sharp corners.

The plate was anchored with four identical springs to a frame as shown if Fig. 10. The frame was rigidly held in the channel by pressing its spiked caps (see in the figure) with the nuts "A" against the concrete walls above the window. Nuts "B" were used to align the anchoring points in the frame with the attachment rings "C" on the plate. The plate was 1cm shorter than the width of the channel, and it was aligned so that a 5mm clearance between the plate and the wall was kept on both sides. The calibration of the anchoring springs showed nonlinear response at small elongations. The bolts "D" were therefore used to stretch the springs to the range of their linear response.

The water depth was 85.7cm throughout all of the tests.

A.4.2. The Multiplate System

The present experimental study with multiplate systems was confined to rigidly held plates. The plates were bolted to a structure made of perforated steel profiles, which were constructed so that wide ranges of drafts and spacings between the plates could be tested. Their width was the same as that of the channel leaving no gaps between the plates and the walls and they protruded high enough above the water to prevent wave overtopping.

The maximum length of the system was 2.5m. The tests were limited to cases of equally spaced plates. Systems of four plates and eight plates were employed (i.e., the maximum spacing for the four plate system was approximately 80cm, and for the eight plates approximately 35cm). The range of plate draft was from 10cm to 30cm.

All the tests were conducted at water depth of 80cm.

Appendix B: Mathematical Expressions

The expressions for the coefficients appearing in eqs. (2.1), (2.2) and (2.3) are as follows:

$$t = \frac{-jK_1}{\pi I_1 - jK_1} \quad ; \quad r = 1 - t$$

$$B_2 = \frac{-2j\mu S_1}{\pi I_1 - jK_1} \quad ; \quad B_4 = \frac{-2jT(S_1 - \pi/4)}{\pi I_1 - jK_1}$$

$$Y_g = \frac{-2gTS_1}{\pi I_1 - jK_1} \quad ; \quad M_g = \frac{-2gT^2(S_1-\pi/4)}{\mu(\pi I_1 - jK_1)}$$

$$\lambda_{22} = \frac{4\sigma T^2 S_1^2}{\pi^2 I_1^2 + K_1^2} \quad ; \quad \lambda_{44} = \frac{4\sigma T^4 (S_1-\pi/4)^2}{\mu^2(\pi^2 I_1^2 + K_1^2)}$$

$$\lambda_{24} = \lambda_{42} = \frac{4\sigma T^3 S_1 (S_1-\pi/4)}{\mu(\pi^2 I_1^2 + K_1^2)}$$

$$\mu_{22} = \frac{4T^2}{\pi} \cdot \{\frac{1}{2} - \frac{S_0}{\mu} + \frac{S_0^{-1}}{\mu^2} - \frac{S_1 \Gamma}{\mu(\pi^2 I_1^2 + K_1^2)}\}$$

$$\mu_{24} = \mu_{42} = \frac{4T^3}{\pi} \cdot \{\frac{\pi}{12} + \frac{1}{2\mu} - \frac{S_0}{\mu^2} + \frac{S_0^{-1}}{\mu^3} - \frac{S_1\Gamma - \pi\Gamma_0/4}{\mu^2(\pi^2 I_1^2 + K_1^2)}\}$$

$$\mu_{44} = \frac{4T^4}{\pi} \{\frac{4+\pi^2}{8\mu^2} + \frac{\pi}{6\mu} + \frac{\pi^2}{64} - (\frac{1}{\mu^3} + \frac{\pi}{4\mu^2})S_0 + \frac{S_0^{-1}}{\mu^4} - \frac{(S_1-\pi/4)(\Gamma/\mu - \pi\mu\gamma_2/4)}{\mu^2(\pi^2 I_1^2 + K_1^2)}\} \quad (*)$$

In the above formulae Γ, Γ_0, and γ_2 are given by:

$$\Gamma = \gamma_1 - \mu\gamma_2 - 0.5\pi K_1 \quad ; \quad \Gamma_0 = \mu^2 S_1 \gamma_2 - \mu S_0(\pi^2 I_1^2 + K_1^2)$$

$$\gamma_1 = \pi^2 I_0^{-1} \cdot I_1 - K_0^{-1} \cdot K_1 \quad ; \quad \gamma_2 = \pi^2 \cdot I_0 \cdot I_1 - K_0 \cdot K_1$$

where $S_0 = 0.5\pi(I_0+L_0)$ and $S_1 = 0.5\pi(I_1+L_1)/\mu$.

L_0, L_1 and K_0, K_1, I_0, I_1 are Struve functions and Bessel functions of the argument $\mu = kT$, respectively. The terms S_0^{-1}, I_0^{-1} and K_0^{-1} are given by

$$S_0^{-1} = \int_0^\mu S_0(\xi)d\xi \ , \quad I_0^{-1} = \int_0^\mu I_0(\xi)d\xi \ , \quad K_0^{-1} = \int_0^\mu K_0(\xi)d\xi$$

(*) The present expression for μ_{44} is slightly different from that in Haskind [1] and Stiassnie [4] due to an algebraic error in those papers.