ELSEVIER

Applied Ocean Research 25 (2003) 263–268

Applied Ocean Research

www.elsevier.com/locate/apor

On a freely floating porous box in shallow water waves

Michael Stiassnie*, Nitai Drimer

Coastal and Marine Engineering Research Institute, Department of Civil and Environmental Engineering, Technion, Haifa 32000, Israel

Received 19 January 2003; accepted 3 December 2003

Abstract

An analytical solution of the flow field, obtained by the interaction of a linear shallow water wave with a freely floating porous box, is derived. The relatively small drift forces obtained in this new solution indicate the advantage in future use of porous structures as floating breakwaters.

© 2004 Elsevier Ltd. All rights reserved.

Keywords: Water waves; Porous structures; Floating breakwaters

1. Introduction

The present note was motivated by the quest for new floating breakwaters. One of the main problems with floating breakwaters are the large oscillatory forces in their mooring systems, see Ref. [1]. To overcome this difficulty, it is practical to consider 'freely floating structures' with very soft moorings, which resist the second-order drift forces, but hardly effect the first-order oscillatory motion. However, even drift forces can be rather large in heavy storms, and affect the survivability of the floating structure.

Here we examine the suitability of freely floating porous structures, which absorb part of the wave energy, to serve as breakwaters; and show that they reduce the drift forces significantly. Dalrymple et al. [2] have studied the interaction of waves with fixed porous boxes, and the present work attempts to extend their long-wave approximation approach to freely floating boxes. In addition to the shallow water assumption, the problem treated herein includes the following two simplifications: (i) the motion of the box is restricted to sway only; and (ii) its draft is equal to the depth of the water.

The formulation is provided in Section 2, followed by the solution in Section 3, and by results and discussion in Section 4.

* Corresponding author.
E-mail address: miky@tx.technion.ac.il (M. Stiassnie).

0141-1187/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.apor.2003.12.001

2. Formulation

We consider the iteraction of a gravity wave train with a porous box of width b, separating two fluid regions of depth h, as shown in Fig. 1.

A wave train, with amplitude a_w, frequency ω and wave-number k, attacks from the left and sets the box into sway motion with amplitude $a_b = \alpha a_w$. For simplicity we assume that the draft of the box is almost h, and neglect its heave and roll motions. We also assume shallow water conditions for which the dispersion relation is

$$\omega^2 = ghk^2 \tag{2.1}$$

The characteristics of the porous medium are its porosity ε, linear friction factor f, and the inertial coefficient s. The broken line in Fig. 1 defines a fixed control surface, which at all times contains the floating box, despite the fact that its sides were chosen close to those of the box. In the sequel, we will use the subscripts 1, 2, and 3 for the domains to the left, within, and to the right of the control volume, respectively. We will also use the subscript $2'$ for a coordinate system oscillating with the box.

The free surface elevations $Re\{a_w \eta_j \, e^{i\omega t}\}$, the horizontal velocities $Re\{a_w \sqrt{g/h} u_j \, e^{i\omega t}\}$, in the regions $j = 1$ and 3, are given by

$$\eta_1 = e^{-ikx} + Re^{ikx} \tag{2.2}$$

$$u_1 = e^{-ikx} - Re^{ikx} \tag{2.3}$$

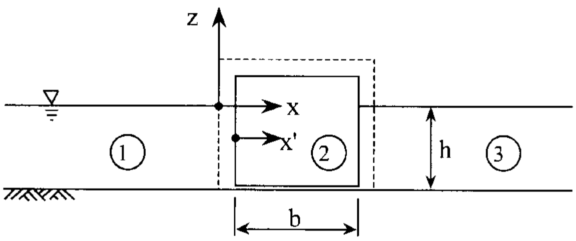

Fig. 1. Schematic diagram.

$$\eta_3 = T\,e^{-ik(x-b)} \tag{2.4}$$

$$u_3 = T\,e^{-ik(x-b)} \tag{2.5}$$

where R and T are the complex reflection and transmission coefficients, respectively.

The scalings of the chosen dimensionless free-surface elevations η_j and horizontal velocities u_j are in accordance to shallow water theory, see Eq. (5.2) in Ref. [3].

Within the porous medium, the water surface elevation $Re\{a_w \eta_2\, e^{i\omega t}\}$, and seepage velocity relative to the moving rigid porous medium $Re\{a_w\sqrt{g/h}q_{2'}\, e^{i\omega t}\}$ are

$$\eta_2 = (s - if)(A_1\, e^{-iKx'} + B_1\, e^{iK(x'-b)}) \tag{2.6}$$

$$q_{2'} = (s - if)^{1/2}(A_1\, e^{-iKx'} - B_1\, e^{iK(x'-b)}) - \frac{i\omega^2\alpha}{gk(s - if)} \tag{2.7}$$

where for shallow water, i.e. small Kh:

$$\omega^2 = ghK^2/(s - if) \tag{2.8}$$

Note that the acceleration of the porous medium is $Re\{-a_b\omega^2\, e^{i\omega t}\}$.

The in and out currents from the porous medium at $x \approx x' = 0$ and $x \approx x' = b$, respectively, as seen by a stationary observer are $Re\{a_w\sqrt{g/h}u_2\, e^{i\omega t}\}$, where

$$u_2 = \varepsilon\left\{(s - if)^{1/2}(A_1\, e^{-iKx} - B_1\, e^{iK(x-b)}) - \frac{i\alpha\omega^2}{gk(s - if)}\right\} + \frac{i\omega^2\alpha}{gk} \tag{2.9}$$

The full derivation of Eqs. (2.6)–(2.9) is outlined in Appendix A.

The continuity of the free-surface elevation (which is equivalent to that of the pressure), and of the mass-flux at the lateral boundaries of the control volume give the following system of four equations:

$$\begin{aligned}
&\eta_1|_{x=0} = 1 + R = (s - if)(A_1 + EB_1) = \eta_2|_{x=0}\\
&u_1\Big|_{x=0} = 1 - R = \varepsilon(s - if)^{1/2}(A_1 - EB_1) + \frac{i\omega^2\alpha}{gk}\left(1 - \frac{\varepsilon}{s - if}\right) = u_2\Big|_{x=0}\\
&\eta_3|_{x=b} = T = (s - if)(EA_1 + B_1) = \eta_2|_{x=b}\\
&u_3\Big|_{x=b} = T = \varepsilon(s - if)^{1/2}(EA_1 - B_1) + \frac{i\omega^2\alpha}{gk}\left(1 - \frac{\varepsilon}{s - if}\right) = u_2\Big|_{x=b}
\end{aligned} \tag{2.10}$$

where

$$E = e^{-iKb} \tag{2.11}$$

Two solutions of Eq. (2.10) are discussed in Section 3.

3. Solutions

3.1. Fixed box

For this case $\alpha \equiv 0$ and Eq. (2.10) have the following solutions:

$$R^{(F)} = \frac{(1 - m^2)(1 - E^2)}{(1 + m)^2 - E^2(1 - m)^2} \tag{3.1}$$

$$T^{(F)} = \frac{4mE}{(1 + m)^2 - E^2(1 - m)^2} \tag{3.2}$$

$$A_1^{(F)} = \frac{2(1 + m)}{(s - if)[(1 + m)^2 - E^2(1 - m)^2]} = \frac{T^{(F)}}{2E(s - if)}\left(1 + \frac{1}{m}\right) \tag{3.3}$$

$$B_1^{(F)} = \frac{-2E(1 - m)}{(s - if)[(1 + m)^2 - E^2(1 - m)^2]} = \frac{T^{(F)}}{2(s - if)}\left(1 - \frac{1}{m}\right) \tag{3.4}$$

where

$$m = \varepsilon/(s - if)^{1/2} \tag{3.5}$$

Eqs. (3.1)–(3.5) are identical to Eqs. (4.2) and (4.3) of Ref. [2], except an error in their expression for B_1, which contains a superfluous E.

3.2. Floating box

The solution of Eq. (2.10) depends on the yet unknown α and is given by

$$R = R^{(F)} - \alpha(1 - E)F \tag{3.6}$$

$$T = T^{(F)} + \alpha(1 - E)F \tag{3.7}$$

$$A_1 = A_1^{(F)} - \frac{\alpha F}{s - if} \tag{3.8}$$

$$B_1 = B_1^{(F)} + \frac{\alpha F}{s - if} \tag{3.9}$$

where

$$F = \frac{i\omega^2}{gk}[1 - \varepsilon/(s - if)]\cdot[(1 + m) - E(1 - m)]^{-1} \tag{3.10}$$

In order to obtain α, we use the equation of motion of the box itself. Denoting the horizontal force (per unit width) applied on the box $Re\{a_w \tilde{R}\, e^{i\omega t}\}$, and taking the mass of the structure to be $(1 - \varepsilon)bh\rho$, we obtain

$$\tilde{R} = -(1 - \varepsilon)bh\rho\omega^2\alpha \tag{3.11}$$

where ρ is the density of the water.

The integral momentum theorem assures that the rate of change of the horizontal momentum of the fluid within the stationary control volume shown in Fig. 1 is equal, to leading order, to the instantaneous forces applied to it. These forces include the pressure forces on the lateral boundaries of the control volume and the horizontal force of the solid structure on the fluid. Note, that the horizontal momentum fluxes through the lateral boundaries of the control volume have not been included, since they are of second-order in amplitude. Thus, the force on the solid structure is given by the difference between the pressure force at $x = 0$ and that at $x = b$, minus the rate of change of the horizontal momentum of the fluid within the control volume.

$$\tilde{R} = \rho g h[\eta_1(0) - \eta_3(b)] - i\omega\rho h\sqrt{\frac{g}{h}} \times \int_0^b \varepsilon\left(q_{2'} + \frac{i\omega^2\alpha}{gk}\right)dx' \tag{3.12}$$

Substituting Eq. (3.12) into Eq. (3.11) yields

$$\alpha = \frac{1 + R^{(F)} - T^{(F)} + \varepsilon(E - 1)(A_1^{(F)} - B_1^{(F)})}{2(1 - E)\left(1 - \frac{\varepsilon}{s - if}\right)F - \frac{\omega^2 b}{g}\left[1 - \frac{\varepsilon}{s - if}\right]} \tag{3.13}$$

which enables to calculate Eqs. (3.6)–(3.9) explicitly.

4. Results and discussion

As a first example, we choose boxes with width to depth ration $b/h = 5$, and assume the following parameters for the porous medium: $\varepsilon = 0.5$, $s = 1$, $f = 1$. In Fig. 2, we show: (a) the transmission coefficient; (b) the reflection coefficient; and (c) the energy dissipation fraction, for three different structures (two freely floating and one fixed). One of the free structures is porous (denoted by – – –) and the other is impermeable (denoted by —). The fixed structure is porous (given by ···).

Substituting $\varepsilon \to 0$ into Eqs. (3.1), (3.2) and (3.13), gives for the impermeable box, the simple expressions:

$$R = 0.5ikb/(1 + 0.5ikb) \tag{4.1}$$

$$T = 1/(1 + 0.5ikb) \tag{4.2}$$

$$\alpha = -i/(kh(1 + 0.5ikb)) \tag{4.3}$$

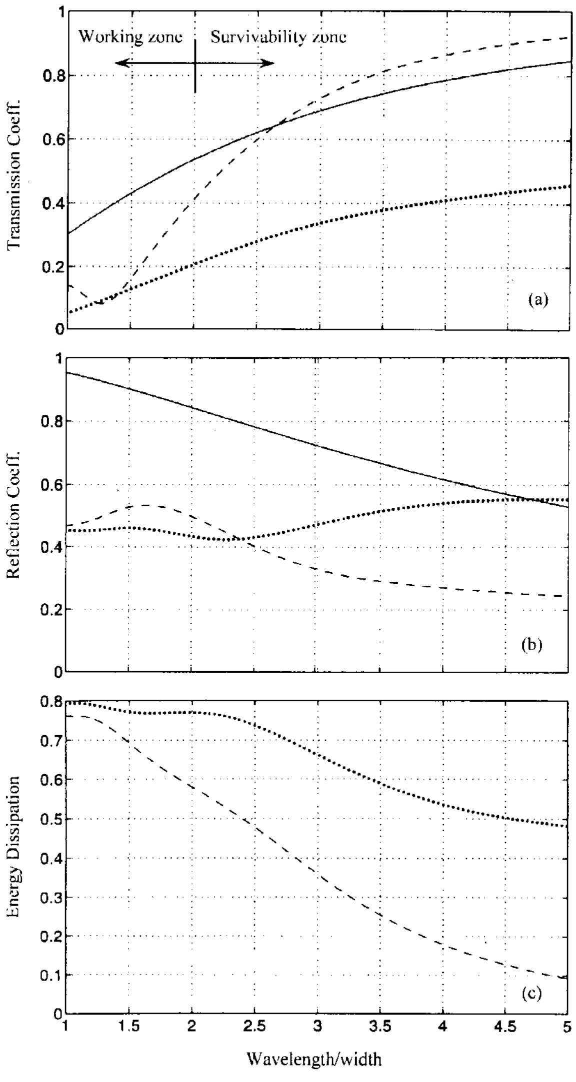

Fig. 2. Comparison of performance between a porous fixed box (···) a porous free box (– – –) and an impermeable free box (—). For $b/h = 5$, $\varepsilon = 0.5$, $s = 1$ and $f = 1$. (a) Transmission coefficient; (b) reflection coefficient; and (c) energy dissipation fraction.

The wavelength to width ration $2\pi/kb$ in all the figures covers the range (1, 5). In the sequel we shall refer to the ranges (1, 2) as 'working zone' and to (2, 5) as 'survivability zone' of the structure. From Fig. 2a, it is seen that the protection provided by the floating porous structure in the working range is better than that of the floating impermeable structure. From Fig. 2b, it is evident that the reflection coefficient for the floating porous structure is significantly smaller than that of the impermeable one. Fig. 2c indicates that more than half of the wave energy is dissipated by the free porous breakwater in its working range, compared to about three-quarters for a fixed porous structure of the same dimensions.

The sway amplitudes for both free structures (see Fig. 3) are almost identical.

Longuet-Higgins [4] provided the following formula for the drift force:

$$D = \frac{\rho g}{4} a_w^2 (1 + |R|^2 - |T|^2)(1 + 2kh/sh(2kh)) \tag{4.4}$$

which for shallow water reduces to

$$D_s = \rho g d/2, \qquad d = 1 + |R|^2 - |T|^2 \tag{4.5}$$

The drift force coefficient d is given in Fig. 4, from which a significant advantage of the porous floating breakwater, compared to the impermeable breakwater, is evident. The drift-force coefficient for the porous floating structure is much smaller than that of the impermeable structure for any wavelength, and in particular in the survivability range. To our opinion, the above results indicate the potential of freely floating porous breakwaters.

As a second example, we investigate the influence of changes in the friction factor f and inertial coefficient s on the structure performance. According to Ref. [2], the inertial coefficient is given by

$$s = 1 + \frac{1 - \varepsilon}{\varepsilon} C_M \tag{4.6}$$

where C_M is the added-mass coefficient of the grain.

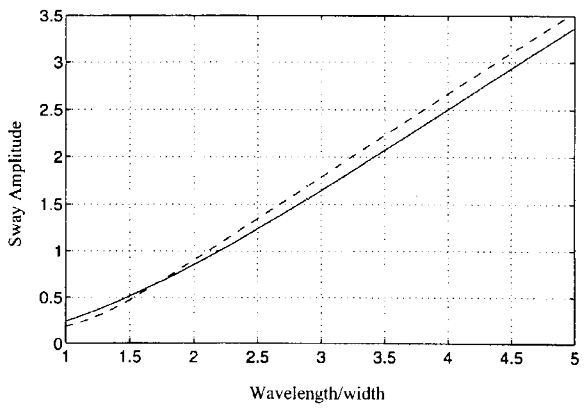

Fig. 3. Ratio of sway amplitude to incident wave amplitude for the same two freely floating boxes as in Fig. 2.

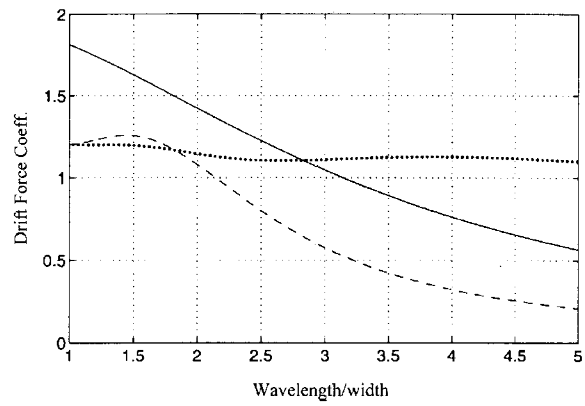

Fig. 4. A comparison of the drift forces between the same three boxes as in Fig. 2.

They also claim that s is often taken as unity in practice, but that some laboratory tests indicate better correlation for values approaching 2. In our second example, we take two values, $s = 1, 2$. For the friction coefficient f, Dalrymple et al. [2] make reference to Lorentz's hypothesis of equivalent work, which enables to calculate f iteratively, if the intrinsic permeability R_p and the turbulent resistance coefficient C_f of the porous medium are known:

$$f = \frac{\omega^{-1} \int_0^b \mathrm{d}x \int_{-h}^0 \mathrm{d}z \int_0^{2\pi/\omega} \varepsilon \left[\frac{\nu}{R_p} + \frac{C_f \varepsilon}{R_p^{1/2}} q_{2'} \right] q_{2'}^2 \mathrm{d}t}{\int_0^b \mathrm{d}x \int_{-h}^0 \mathrm{d}z \int_0^{2\pi/\omega} q_{2'}^2 \mathrm{d}t} \tag{4.7}$$

where ν is the kinematic viscosity of the water.

However, Dalrymple et al. [2] take f as a given constant, and add, that although in principle, its values could range from zero to infinity, for porous breakwaters it is of O(1). In our second example, we take two values, $f = 1, 10$. In Fig. 5, we show: (a) the transmission coefficient; (b) the reflection coefficient; and (c) the energy dissipation fraction, for three different porous freely floating structures. One is the same as in the first example, i.e. with $s = 1$ and $f = 1$ (denoted by – – –), another is with $s = 1$, but $f = 10$ (denoted by ···) and the third is for $s = 2$ and $f = 1$ (denoted by —). The sway amplitudes, and the drift force coefficients for these structures, are given in Figs. 6 and 7, respectively.

The influence of the increase in f: The increase of f reduces the dissipation and increases the reflection in almost the whole wavelength/width range. The transmission is increased in the working-range, but reduced in the survivability range. The largest transmission coefficient in the working range for $f = 10$ is 0.47, which means that only about 20% of the wave energy reaches the protected zone. The increase of f from 1 to 10 has no significant effect on the sway of the box (see Fig. 6), but increases the maximum drift force by 25%. Generally speaking, a structure with f closer to 1 has a favorable performance compared to one with $f = 10$.

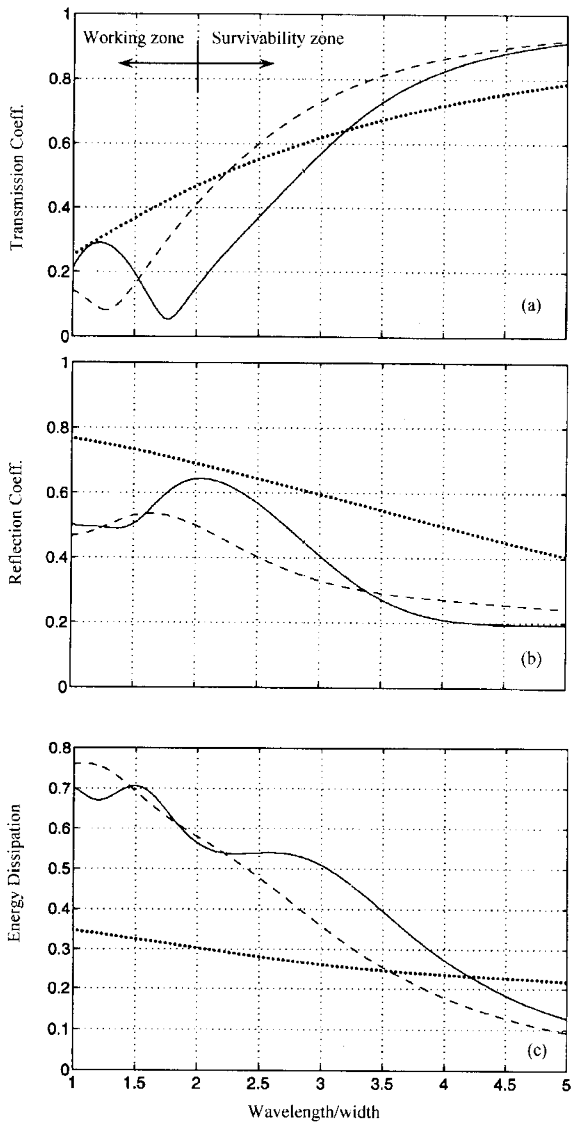

Fig. 5. Comparison of performance between three different porous freely floating boxes, all with $b/h = 5$ and $\varepsilon = 0.5$: (a) $s = 1, f = 1$ (– – –); (b) $s = 1, f = 10$. (···); and (c) $s = 2, f = 1$ (—).

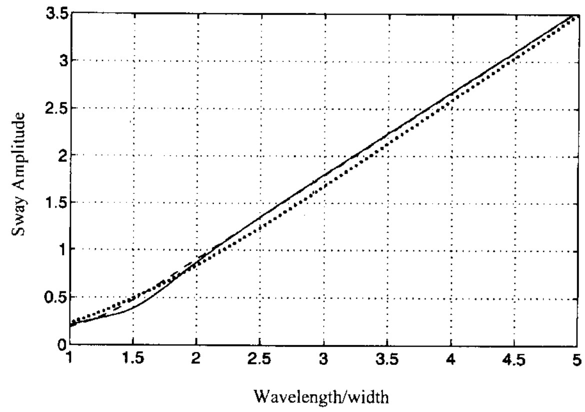

Fig. 6. Ratio of sway amplitude to incident wave amplitude for the same three freely floating boxes as in Fig. 5.

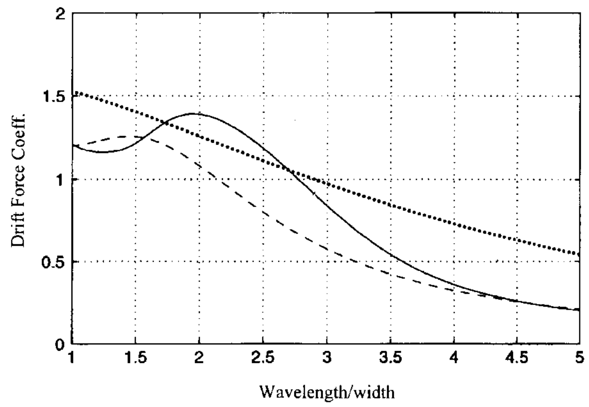

Fig. 7. A comparison of the drift forces between the same three boxes as in Fig. 5.

The influence of the increase in s: From Fig. 5a, we see that the structure with $s = 2$ has a wider working range (if defined as the wavelength/width range for which the transmission coefficient is smaller than 0.4). The dissipation fraction is similar for both s values (see Fig. 5c), the sway is hardly changed (Fig. 6), but the drift force increases.

The influence of the increase in ε: Computations with $s = f = 1$, but with $\varepsilon = 0.9$ give transmission coefficients similar to those for $\varepsilon = 0.5$, and substantially reduced reflection coefficients. The latter is related to higher energy dissipation fractions and to a 20% reduction in the drift force coefficient over both the working and survivability zones.

Appendix A. Equations in the porous medium

According to Appendix A of Ref. [2], the basic field equations in a porous medium at rest are the conservation of mass:

$$\operatorname{div} \vec{q} = 0, \tag{A.1}$$

and the linearized equation of motion

$$s\frac{\partial \vec{q}}{\partial t} = -\nabla\left(\frac{p}{\rho} + gz\right) - \omega f \vec{q} \tag{A.2}$$

where $\vec{q}$ is the seepage velocity vector and p is the pressure. As before, s is the inertial coefficient, and f a friction factor.

The above are supplemented by the bottom and phreatic surface boundary conditions. For an oscillating medium, Eq. (A.1) remains valid, and a fictitious body force per unit mass (underlined in Eq. (A.3)) is added to Eq. (A.2).

$$s\frac{\partial \vec{q}}{\partial t} = -\nabla\left(\frac{p}{\rho} + gz\right) - \omega f \vec{q} + \underline{\nabla(\omega^2 a_b\, e^{i\omega t} x)} \tag{A.3}$$

taking the curl of Eq. (A.3), gives curl $\vec{q} = 0$, i.e. $\vec{q} = \nabla\Phi$, so that Eq. (A.1) yields

$$\nabla^2\Phi = 0 \tag{A.4}$$

Eq. (A.3) is integrated to give

$$s\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + gz + f\omega\Phi - \omega^2 a_b\, e^{i\omega t} x = 0 \tag{A.5}$$

Note that the boundary conditions in terms of Φ are:

$$\frac{\partial \Phi}{\partial z} = 0; \quad \text{at } z = -h \tag{A.6}$$

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \eta}{\partial t} \text{ and } p = 0; \quad \text{at } z = 0. \tag{A.7}$$

Combining Eqs. (A.7) and (A.5) give at $z = 0$:

$$s\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} + f\omega\frac{\partial \Phi}{\partial t} - i\omega^3 a_b x\, e^{i\omega t} = 0 \tag{A.8}$$

The solution of Eqs. (A.4), (A.6), and (A.8) is

$$\Phi = \frac{i g a_w}{\omega}\frac{\cosh[K(z+h)]}{\cosh(Kh)}\{A_1\, e^{-iKx} + B_1\, e^{iK(x-b)}\}e^{i\omega t} - \frac{i\omega a_b x\, e^{i\omega t}}{s - if} \tag{A.9}$$

where K is given by Eq. (2.8).

The horizontal component of the seepage velocity, as seen by an observer fixed to the porous medium is

$$\frac{\partial \Phi}{\partial x} = \frac{g a_w K}{\omega}\frac{\cosh[K(z+h)]}{\cosh(Kh)}\{A_1 e^{-iKx} - B_1\, e^{iK(x-b)}\}e^{i\omega t} - \frac{i\omega a_b}{s - if}e^{i\omega t} \tag{A.10}$$

which for small Kh yields $q_{2'}$ in Eq. (2.7).

Substituting Eq. (A.9) into Eq. (A.7) and integrating in time gives Eq. (2.6). Moreover, u_2, the current through the control surfaces ($x = 0$, $x = b$) in the laboratory frame of reference is

$$\varepsilon\frac{\partial \Phi}{\partial x} + i\omega a_b = \varepsilon\left[\frac{g a_w K}{\omega}\{A_1\, e^{-iKx} - B_1\, e^{iK(x-b)}\} - \frac{i\omega a_b}{s - if}\right]e^{i\omega t} + i\omega a_b\, e^{i\omega t} \tag{A.11}$$

which yields Eq. (2.9).

References

[1] Drimer N, Agnon Y, Stiassnie M. A simplified analytical model for a floating breakwater in water of finite depth. Appl Ocean Res 1992;14: 33–41.

[2] Dalrymple RA, Losada MA, Martin PA. Reflection and transmission from porous structures under oblique wave attack. J Fluid Mech 1991; 224:625–44.

[3] Dean RG, Dalrymple RA. Water wave mechanics for engineers and scientists. Singapore: World Scientific; 1991. p. 353.

[4] Longuet-Higgens MS. The mean forces exerted by waves on floating or submerged bodies with applications to sand bars and wave power machines. Proc R Soc Lond A 1997;352:463–80.