Applied Ocean Research 14 (1992) 33–41

A simplified analytical model for a floating breakwater in water of finite depth

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(Received 7 July 1991; accepted 17 July 1991)

The performance of a box-type floating breakwater is studied. The implementation of simplifying assumptions concerning the flow beneath a pontoon-type floating breakwater, leads to an analytical solution of the two-dimensional linearized hydrodynamic problem. Comparison of the analytical results with a numerical solution of the full linear problem shows good agreement over a wide range of parameters.

1 INTRODUCTION

Floating breakwaters (FBs) are increasingly used in the protection of small boat marinas.[1,2] One of the most common FB types in use today is the pontoon or box-type which has proven performance and is effective in moderate wave conditions.[2] The two-dimensional linearized hydrodynamic problem of a box-type structure has been solved using various numerical methods.[3,4,5,6,7]

This paper presents an analytical solution for a simplified problem, where all the interesting results: added mass and damping coefficients, mooring forces, transmission and reflection coefficients, are given by simple analytical expressions.

Comparison with a numerical solution of the full linear problem shows good agreement for a large range of parameters. The effects of the FB's dimensions and mooring stiffness in the case of finite water depth are studied and several practical conclusions are presented. Notably, the pontoon-type FB is found to perform very well in water of intermediate depth.

2 FORMATION OF THE MATHEMATICAL PROBLEM

We consider a long pontoon with a rectangular cross-section. Let $2B$ be its breadth and d its draft at rest. The FB is free to oscillate in three modes of motion: sway, heave and roll. A Cartesian coordinate system is chosen with the origin and the x–y-coordinates in the undisturbed free surface, and the z-axis points vertically upwards (see Fig. 1). The water depth is h and the clearance between the FB's bottom and the sea bed is $S (S = h\text{-}d)$.

As usual the flow is assumed to be inviscid, irrotational and periodic, and the velocity potential is

$$\Phi(y, z, t) = \mathrm{Re}\,[\phi(y, z)\mathrm{e}^{-\mathrm{i}\omega t}] \tag{1}$$

where $\mathrm{i} = \sqrt{-1}$, t is the time and ω is the angular frequency.

The time-independent potential $\phi(y, z)$ satisfied the following boundary-value problem:

Laplace equation:

$$\nabla^2\phi = 0 \quad \text{in the flow domain} \tag{2}$$

Free surface condition:

$$\frac{\partial\phi}{\partial z} = \sigma\phi \quad \text{on } z = 0, |y| > B, \text{ where } \sigma = \frac{\omega^2}{g} \tag{3}$$

Sea-bed condition:

$$\frac{\partial\phi}{\partial z} = 0 \quad \text{on } z = h \tag{4}$$

Rigid body condition:

$$\frac{\partial\phi}{\partial n} = \mathbf{V}_\mathrm{B}\cdot\mathbf{n} \quad \text{on the body surface} \tag{5}$$

where $\mathbf{V}_\mathrm{B}$ is the body's velocity vector and $\mathbf{n}$ is a unit normal to the body surface pointing out of the fluid, and appropriate radiation conditions apply.

Following Newman's[8] notation the potential may be decomposed into four parts:

$$\Phi(y, z, t) = \mathrm{Re}\,\{[\phi_7(y, z) + \sum_{i=2}^{4} X_i\phi_i(y, z)]\mathrm{e}^{-\mathrm{i}\omega t}\} \tag{6}$$

Applied Ocean Research 0141-1187/92/$05.00

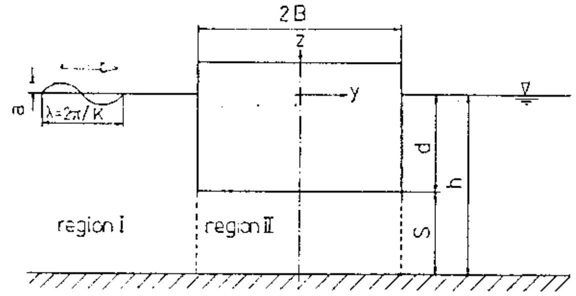

Fig. 1. Schematic illustration of a rectangular cross-section floating breakwater.

where ϕ_7 is the time-independent potential due to an incident wave scattered by a fixed body and ϕ_i ($i = 2, 3, 4$) are the radiated potentials caused by unit amplitude sway, heave and roll oscillations of the FB, respectively. X_i is the complex amplitude of the body's response motion in the ith mode. Each of the four sub-problems is solved separately.

The body responses X_i are found from the equations of motion of the FB, where all four modes interact with each other.

3 ANALYTICAL SOLUTION

In region II the flow potential function is the sum of a particular solution satisfying eqns (2), (4) and (5) and a homogeneous solution satisfying

$$\left\{\begin{array}{ll} \dfrac{\partial^2\phi}{\partial y^2} + \dfrac{\partial^2\phi}{\partial z^2} = 0 & -B \leqslant y \leqslant B,\ -h \leqslant z \leqslant -d \\ \dfrac{\partial\phi}{\partial z} = 0 & \text{on } z = -h \text{ and on } z = -d \end{array}\right\} \tag{7}$$

Since the FB has a vertical symmetry plane, it is convenient to deal separately with flows that are symmetric or antisymmetric about $y = 0$. The flows caused by sway or roll are antisymmetric, while the heave flow is symmetric. The scattering flow may be decomposed into symmetric and antisymmetric components; each can be thought of as caused by two incident waves, one from the right and one from the left side of the FB. The two waves have the same length and amplitude and are phase-symmetric or antisymmetric in the symmetric or antisymmetric scattering flows, respectively.

A general solution of eqn (7) will be of the form

$$\phi^{\binom{S}{A}} = \begin{Bmatrix} A_0 \\ B_0 y \end{Bmatrix} F_0 + \sum_{n=1}^{\infty} \begin{Bmatrix} A_n \\ B_n \end{Bmatrix} F_n \begin{Bmatrix} \cosh(Q_n y) \\ \sinh(Q_n y) \end{Bmatrix} \tag{8}$$

for symmetric (S) or antisymmetric (A) flows respectively. F_n ($n = 0, 1, \ldots$) is a complete set of orthonormal functions in the interval $(-h, 0)$ given by:

$$F_0 = S^{-1/2}, \quad F_n = \sqrt{2}\, S^{-1/2} \cos\,[Q_n(z + h)],$$

$$Q_n = \frac{n\pi}{S}, \ (n = 1, 2, \ldots) \tag{9}$$

For the limiting case of a long wave and a narrow gap, in which the FB breadth ($2B$), and the wavelength (λ), are taken to be much greater than the gap below (S), we may deduce from eqn (7), by scaling arguments, a simplified solution which is the first expression on the right-hand side of eqn (8), namely, a constant for symmetric flows and a uniform flow potential in antisymmetric flows.

Before the presentation of a detailed solution for each of the sub-problems, we write the principal stages:

(a) Writing a general solution for the flow potential in region I as an eigenfunction expansion satisfying eqns (2), (3) and (4).
(b) Writing an approximate solution, for the flow potential in region II, which consists of an appropriate particular solution and an unknown constant potential or constant horizontal velocity as described above.
(c) Applying the horizontal velocity continuity condition at $y = -B$, from which we express the unknown coefficient of the eigenfunctions expansion in region I in terms of the one unknown constant of the flow potential in region II.
(d) Finding the unknown constant from an integral momentum balance on the fluid in region II.
(e) After the flow potential is known, we can evaluate the various quantities of interest as simple analytical expressions.

Note that the second expression on the right-hand side of eqn (8) contains modes which cause neither mass flux nor momentum flux through the interface between the two regions ($y = -B$, $-h < z < -d$); hence we expect the effect of truncating these terms on the far-field properties to be minor.

Another way to check the validity of the approximate solution is to compute the horizontal velocity on $y = -B$, $-h < z < d$, from the full linear theory, and assess its uniformity.

For the dimensions $B = h$, $S = 0{\cdot}3h$, ($2B/S \cong 7$) the horizontal velocity amplitude variation is $\dfrac{U_{max} - U_{min}}{U_{max}} \cong 25\%$ for an incident wavelength $\lambda/h = 8$ and about 18% for $\lambda/h = 12$. The phase variation is less than 2% in both cases.

We will see that for the quantities of interest, such as hydrodynamic coefficients, exciting forces and transmission coefficients, the resulting formulas predict values which agree well with the correct ones through the full range of wave numbers computed.

3.1 The scattering problem

As was stated above, it is convenient to decompose the scattering flow potential into symmetric and antisymmetric components. A general solution in region I which satisfied the Laplace equation, free surface and the sea-bed boundary conditions will be of the form

$$\Phi = \mathrm{Re}\,[(\phi^{(S)} + \phi^{(A)})e^{-i\omega t}],$$

$$\phi^{(\mathrm{S})}_{(\mathrm{A})} = \begin{bmatrix} a_{7,0} \\ b_{7,0} \end{bmatrix} f_0[e^{ik(y+B)} + R^{(\mathrm{S})}_{(\mathrm{A})} e^{-ik(y+B)}] + \sum_{n=1}^{\infty} \begin{pmatrix} a_{7n} \\ b_{7,n} \end{pmatrix} f_n e^{k_n(y+B)}, y \leqslant -B \quad (10)$$

By the definition of symmetry and antisymmetry about $y = 0$, the flow potential to the right of the FB will take the form

$$\Phi(+y, z, t) = \mathrm{Re}\,\{[\phi^{(S)}(-y, z) - \phi^{(A)}(-y, z)]e^{-i\omega t}\}, \quad y \geqslant B \quad (11)$$

f_n $(n = 0, 1, \ldots)$ is a complete set of orthonormal functions in the interval $(-h, 0)$ given by

$$f_0 = \frac{\sqrt{2}\cosh[k(z + h)]}{[h + \sigma^{-1}\sinh^2(kh)]^{1/2}},$$

$$f_n = \frac{\sqrt{2}\cos[k_n(z + h)]}{[h - \sigma^{-1}\sin^2(k_n h)]^{1/2}}, (n = 1, 2, \ldots) \quad (12)$$

k is the incident wave number which satisfies the dispersion relation

$$\sigma = k\tanh(kh) \quad (13)$$

k_n are the positive roots of the equation

$$\sigma = -k_n \tan(k_n h) \quad (14)$$

It is easy to see from eqns (10) and (11) that in the far field, $|y| \rightarrow \infty$, we are left with propagating waves which have the following complex amplitudes:

To the left of the FB
$a_{7,0} + b_{7,0}$ correspond to the incident wave traveling to the right,
$a_{7,0}R^{(S)} + b_{7,0}R^{(A)}$ correspond to the reflected wave traveling to the left.
To the right of the FB:
$a_{7,0}R^{(S)} + b_{7,0}R^{(A)}$ correspond to the transmitted wave traveling to the right.
$a_{7,0} + b_{7,0}$ correspond to a wave which is traveling to the left.

The last wave does not satisfy the radiation condition and hence must vanish.

Using the linear relation between the flow potential and the sea surface function:

$$\eta = -g\frac{\partial\Phi}{\partial t} \quad \text{on } z = 0$$

we have

$$a_{7,0} = b_{7,0} = \frac{-iag}{2\omega f_0(0)} \quad (15)$$

where a is the amplitude of the incident wave. The fixed body reflection and transmission coefficients are given by

$$\begin{pmatrix} R \\ T \end{pmatrix} = \tfrac{1}{2}(R^{S} \pm R^{A}) \quad (16)$$

According to our assumption, the velocity potential in region II will have the form

$$\Phi = \mathrm{Re}\,[(vy + \phi_0)e^{-i\omega t}], \; -B \leqslant y \leqslant 0 \quad (17)$$

where ϕ_0, v are complex constants.

We apply the horizontal velocity continuity condition at $y = -B$

$$\frac{\partial}{\partial y}\phi(y = -B) = \begin{Bmatrix} 0 & -d < z < 0 \\ v & -h < z < -d \end{Bmatrix} \quad (18)$$

Using the orthonormality of $\{f_n\}$ we can express the unknown constants in eqn (7) in terms of v:

$$R^{S} = 1, a_{7,n} = 0 \; (n = 1, 2, \ldots) \quad (19)$$

$$R^{A} = 1 + \frac{iU_0 v}{ka_{7,0}}, b_{7,n} = \frac{U_n v}{k_n}, (n = 1, 2, \ldots) \quad (20)$$

where

$$U_n = \int_{-h}^{-d} f_n \mathrm{d}z \quad (n = 0, 1, 2, \ldots) \quad (21)$$

The unknown, v, will be found from an integral momentum balance on the fluid in region II:

$$F = \int_{-h}^{-d} [P(-B, z) - P(B, z)]\,\mathrm{d}z = m\dot{V},$$

$$m = 2\rho BS \quad (22)$$

where P is the hydrodynamic pressure given by the linearized Bernoulli's equation

$$P = -\rho\frac{\partial\Phi}{\partial t} \quad (23)$$

Substituting eqns (19), (20), (10) and (23) in (22) we get

$$v = -\frac{2a_{7,0}U_0}{SB + \dfrac{iU_0^2}{k} + \sum\limits_{n=1}^{\infty}\dfrac{U_n^2}{k_n}} \quad (24)$$

The fixed body transmission and reflection coefficients can now be calculated using eqns (16), (19), (20) and (24):

$$T = \frac{iU_0^2/k}{SB + iU_0^2/k + \sum\limits_{n=1}^{\infty}\dfrac{U_n^2}{k_n}} \quad (25)$$

$$R = 1 - T$$

Note that the FB's breadth appears only once in the expression for the transmission coefficient and it can be seen that the physical meaning of expanding the FB's breadth is increasing the inertia of the fluid beneath the FB's bottom.

The horizontal force and the moment about the center of flotation, $(y, z) = (0, 0)$ are calculated by integrating the hydrodynamic pressure (eqn (23)) along the sides of the FB; the resulting expressions are

Horizontal force:

$$F_{72} = \mathrm{Re}\,(f_{72}\mathrm{e}^{-i\omega t}),$$
$$f_{72} = 4a_{7,0}i\omega\rho\left[W_0 - T\left(W_0 - \frac{ik}{U_0}\sum_{n=1}^{\infty}\frac{U_n W_n}{k_n}\right)\right] \quad (26)$$

Moment:

$$F_{74} = \mathrm{Re}\,(f_{74}\mathrm{e}^{-i\omega t}),$$
$$f_{74} = 4i\omega\rho a_{7,0} \times\left\{-\tilde{W}_0 + T\left[\tilde{W}_0 - \frac{ik}{U_0}\left(\sum_{n=1}^{\infty}\frac{U_n\tilde{W}_n}{k_n} - \frac{B^3}{3}\right)\right]\right\} \quad (27)$$

where

$$W_n = \int_{-d}^{0} f_n \mathrm{d}z, \quad \tilde{W}_n = \int_{-d}^{0} z f_n \mathrm{d}z \quad (28)$$

In order to calculate the vertical force, we have to find the symmetric potential in region II (which is the constant ϕ_0 in eqn (17)); we apply an integral law of action and reaction to the section $-h \leqslant z \leqslant d$ at $y = -B$, which requires that the total forces acting on the left and right sides of the section will be of the same magnitude and opposite directions:

$$\int_{-h}^{-d}\phi^{\mathrm{I}}\mathrm{d}z = \int_{-h}^{-d}\phi^{\mathrm{II}}\mathrm{d}z \quad \text{on } y = -B \quad (29)$$

substituting ϕ^{I} and ϕ^{II} from eqns (10) and (17), respectively, gives

$$\phi_0 = \frac{2a_{7,0}U_0}{S} \quad (30)$$

Integrating the hydrodynamic pressure, eqn (23), on the FB's bottom, we find the vertical force:

$$F_{73} = \mathrm{Re}\,(f_{73}\mathrm{e}^{-i\omega t}), \quad f_{73} = 2i\omega\rho\, Ba_{7,0}U_0/S \quad (31)$$

3.2 The sway problem

The sway potential is antisymmetric and the velocity potential in region I will be of the form:

$$\phi = b_{2,0}f_0\mathrm{e}^{-ik(y+B)} + \sum_{n=1}^{\infty} b_{2,n}f_n\mathrm{e}^{k_n(y+B)}, \ y < -B \quad (32)$$

In region II we assume a uniform and horizontal flow potential: $\phi = vy$.

Applying horizontal velocity continuity on $y = -B$ we can express $b_{2,0}$ and $b_{2,n}$ in terms of v:

$$b_{2,0} = \frac{i}{k}vU_0 + \frac{\omega}{k}W_0, \ b_{2,n} = \frac{1}{k_n}(vU_n - i\omega W_n) \quad (33)$$

applying an integral momentum balance to the fluid in region II, eqn (23), and solving for v we obtain

$$v = \omega\left(i\sum_{n=1}^{\infty}\left(\frac{U_nW_n}{k_n} - \frac{U_0W_0}{k}\right)\right)\Big/\left(BS + \frac{iU_0^2}{k} + \sum_{n=1}^{\infty}\frac{U_n^2}{k_n}\right) \quad (34)$$

The hydrodynamic horizontal force due to sway motion is then found by integrating the hydrodynamic pressure, eqn (23), on the FB sides:

$$F_{22} = \mathrm{Re}\,(f_{22}\mathrm{e}^{-i\omega t}), f_{22} = 2i\omega\rho\left(b_{2,0}W_0 + \sum_{n=1}^{\infty} b_{2,n}W_n\right) \quad (35)$$

and the moment about the center of flotation will be of the form

$$F_{24} = \mathrm{Re}\,(f_{24}\mathrm{e}^{-i\omega t}),$$
$$f_{24} = 2i\omega\rho\left[-\left(b_{2,0}\tilde{W}_0 + \sum_{n=1}^{\infty} b_{2,n}\tilde{W}_n\right) + v\frac{B^3}{3}\right] \quad (36)$$

The general forces can be decomposed into inertia and damping parts:

$$f_{ij} = \omega^2 a_{ij} + i\omega b_{ij} \quad (37)$$

where a_{ij} and b_{ij}, both real quantities, are known as the added mass and damping coefficients.

3.3 The roll problem

In region I the antisymmetric velocity potential caused by roll will be of the form:

$$\phi = b_{4,0}f_0\mathrm{e}^{-ik(y+B)} + \sum_{n=1}^{\infty} b_{4,n}f_n\mathrm{e}^{k_n(y+B)} \quad (38)$$

In region II we add a uniform and horizontal flow potential to a particular solution of eqn (2) satisfying eqn (4) and the body boundary condition

$$\frac{\partial\phi}{\partial z} = -i\omega y, \ z = -d \quad (39)$$

This results in a flow potential of the form

$$\phi = \frac{i\omega y}{6S}[y^2 + 3h^2 - 3(h+z)^2] + Cy \quad (40)$$

where C is an unknown complex constant.

From horizontal velocity continuity on $y = -B$ we express $b_{4,0}$ and $b_{4,n}$ in terms of C:

$$b_{4,0} = A_0 + \frac{iU_0C}{k}, \ b_{4,n} = A_n + \frac{U_nC}{k_n} \quad (41)$$

where

$$A_0 = -\frac{\omega}{k}\left\{\frac{1}{2S}[(B^2 + h^2)U_0 - \tilde{U}_0] + \tilde{W}_0\right\},$$
$$A_n = \frac{i\omega}{k_n}\left\{\frac{1}{2S}[(B^2 + h^2)U_n - \tilde{U}_n] + \tilde{W}_n\right\} \quad (42)$$

and

$$\tilde{U}_n = \int_{-h}^{-d} (z + h)^2 f_n \mathrm{d}z \tag{43}$$

the constant C is found from the integral law of action and reaction (eqn (29)):

$$C = -\left[\frac{i\omega B}{6}(B^2 + 3h^2 - S^2) + A_0 U_0 + \sum_{n=1}^{\infty} A_n U_n\right] \Big/ \left(\frac{iU_0^2}{k} + \sum_{n=1}^{\infty} \frac{U_n^2}{k_n} + BS\right) \tag{44}$$

The hydrodynamic horizontal force caused by roll is

$$F_{24} = \mathrm{Re}\,(f_{24}\mathrm{e}^{-i\omega t}), f_{24} = 2i\omega\rho\,(b_{4,0} W_0 + \sum_{n=1}^{\infty} b_{4,n} W_n) \tag{45}$$

and the moment about the center of flotation is:

$$F_{44} = \mathrm{Re}\,(f_{44}\mathrm{e}^{-i\omega t}),$$

$$f_{44} = 2i\omega\rho\left[-\left(b_{4,0}\tilde{W}_0 + \sum_{n=1}^{\infty} b_{4,n}\tilde{W}_n\right) + \frac{i\omega B^3}{6S}\left(\frac{B^2}{S} + 2hd - d^2\right) + C\frac{B^3}{3}\right] \tag{46}$$

3.4 The heave problem

The heave problem is symmetric about the z-axis, and according to our simplifying assumption the flow potential in region II is the sum of an unknown constant ϕ_0, and a particular solution satisfying the Laplace equation, (eqn (2)), sea-bed condition (eqn (4)) and body boundary condition

$$\frac{\partial\phi}{\partial z} = -i\omega y, \; z = -d \tag{47}$$

Thus we obtain

$$\phi = -\frac{i\omega}{2S}(z^2 + 2hz - y^2) + \phi_0, \quad -B \leqslant y \leqslant 0 \tag{48}$$

In region I we have the symmetric potential of the form

$$\phi = a_{3,0} f_0 \mathrm{e}^{-ik(y+B)} + \sum_{n=1}^{\infty} a_{3,n} f_n \mathrm{e}^{k_n(y+B)}, \; y \leqslant -B \tag{49}$$

Applying horizontal velocity continuity on $y = -B$, we find $a_{3,0}$ and $a_{3,n}$ to be

$$a_{3,0} = \frac{\omega B U_0}{kS}, \; a_{3,n} = -\frac{i\omega B}{k_n S} U_n \tag{50}$$

the constant ϕ_0 is found by satisfying the integral law of action and reaction, eqn (29), on the section $y = -B$:

$$\phi_0 = \frac{\omega B}{kS^2} U_0^2 - \frac{i\omega B}{S^2} \sum_{n=1}^{\infty} \frac{U_n^2}{k_n} + \frac{i\omega}{2S}\left(\frac{S^2}{3} - h^2 - B^2\right) \tag{51}$$

Integrating the hydrodynamic pressure, eqn (23), on the bottom of the FB, we find the vertical force due to the heave motion

$$F_{33} = \mathrm{Re}\,(f_{33}\mathrm{e}^{-i\omega t}),$$

$$f_{33} = \omega^2\left[\rho\frac{2B}{S}\left(\frac{B}{S}\sum_{n=1}^{\infty}\frac{U_n^2}{k_n} + \frac{B^2 + S^2}{3}\right)\right] + i\omega\left(\rho\frac{2\omega B^2}{kS^2}U_0^2\right) \tag{52}$$

the heave added mass and damping coefficients are defined by eqn (37) which, by comparison with eqn (52), yields

$$a_{33} = \rho\frac{2B}{S}\left(\frac{B}{S}\sum_{n=1}^{\infty}\frac{U_n^2}{k_n} + \frac{B^2 + S^2}{3}\right),$$

$$b_{33} = \rho\frac{2\omega B^2}{kS^2}U_0^2 \tag{53}$$

3.5 The combined problem

After solving the four sub-problems, amplitudes of the body response X_2, X_3, X_4 in sway, heave and roll, respectively, are found from the FB's equations of motion:

Horizontal motion:

$$[-\omega^2(m + a_{22}) - i\omega b_{22} + S_2]X_2 - (\omega^2 a_{24} - i\omega b_{24})X_4 = af_{72} \tag{54}$$

Vertical motion:

$$[-\omega^2(m + a_{33}) - i\omega b_{33} + 2\rho g B]X_3 = af_{73} \tag{55}$$

Angular motion:

$$[-\omega^2 a_{42} - i\omega b_{42} + (\overline{\mathrm{KG}} - d)m\omega^2]X_2 + [-\omega^2(m_2 + a_{44}) - i\omega b_{44} + mg\overline{\mathrm{GM}}]X_4 = af_{74} \tag{56}$$

where S_2 is a linear spring constant modeling the mooring system, $\overline{\mathrm{KG}}$ is the elevation of the center of gravity of the FB above the keel, $\overline{\mathrm{GM}}$ is the FB's metacentric arm, namely: the arm of the righting moment at small list, m is the FB's mass ($m = 2Bd\rho$), and m_2 is the FB's moment of intertia about the center of flotation.

The influence of the mooring forces on the vertical and angular motions is neglected in eqns (55–56) since it is usually small compared to the hydrostatic restoring forces.

The transmission and reflection coefficients of the FB problem are

$$T_{\mathrm{FB}} = \frac{i\omega}{g}(2a_{7,0}T - X_2 b_{2,0} + X_3 a_{3,0} - X_4 b_{4,0}) f_0(0)\mathrm{e}^{-2ikB} \tag{57}$$

$$R_{\mathrm{FB}} = \frac{i\omega}{g}(2a_{7,0}T + X_2 b_{2,0} + X_3 a_{3,0} + X_4 b_{4,0}) f_0(0) \tag{58}$$

The horizontal drift force is given by Longuet-Higgins[9] as

$$F_{\mathrm{D.C.}} = \frac{\rho g}{4}(1 + |R|^2 - |T|^2)\left[1 + \frac{2kh}{\sinh(2kh)}\right] \tag{59}$$

4 CONSISTENCY CHECKS

4.1 Confirmation by Haskind's theorem

According to Haskind's theorem (see Mei[10]) the amplitudes of the waves radiated by sway, heave and roll motions are related to the horizontal, vertical and angular exciting forces on a fixed body by the following expressions:

$$f_{72} = 4ik\rho\, a_{7,0}b_{2,0} \tag{60}$$

$$f_{73} = 4ik\rho\, a_{7,0}a_{3,0} \tag{61}$$

$$f_{74} = 4ik\rho\, a_{7,0}b_{4,0} \tag{62}$$

Substituting eqns (26) and (33) in eqn (60) or (31) and eqn (50) in eqn (61), it is easy to see that the Haskind relations for sway and heave are identically satisfied.

Substituting eqns (27) and (41) in eqn (62), it can be shown that eqn (62) is not identically satisfied, but to the order $(kS)^4$. Hence the Haskind relation concerning roll is approximately satisfied.

4.2 Comparison with numerical computations

The full linearized problem (without the implementation of simplifying assumptions in region II) was solved by the method of eigenfunction matching. The solution in each region is expressed as a sum of eigenfunctions with unknown coefficients which are then found by satisfying velocity and pressure continuity at the region's interface ($y = -B$). Mei and Black[11] used this method to solve the scattering (by a fixed body) problem. Black *et al.*[12] solved the general two-dimensional radiation problem. The results obtained were the far-field properties only, from which the wave forces on a stationary body were calculated using Haskind's theorem. Agnon *et al.*[13] solved the scattering and the sway sub-problems. A full development of the solution and numerical results which include also the hydrodynamic coefficients in the three modes of oscillation are given by Drimer.[14]

Forty eigenfunctions were used in region I and 10 in region II in order to get accuracy better than 1%. The finite sums appearing in the analytical solution expressions were truncated after 40 terms.

Before comparing numerical results of the approximate method with the method of eigenfunctions matching, we would like to make a theoretical comparison between the two. Both of the methods use eqn (10) as a general solution of eqns (2), (3) and (4) in region I.

In the exact method, the sum of a homogeneous solution, satisfying eqn (7), and appropriate particular solution, satisfying eqns (2), (4) and (5), is the general solution in region II, using horizontal velocity continuity between the two regions, the coefficients of the eigenfunctions expansion in region I are expressed in terms of the general solution in region II, and then a pressure continuity is written from which a system of linear equations with the unknown coefficients of the solution in region II is obtained.

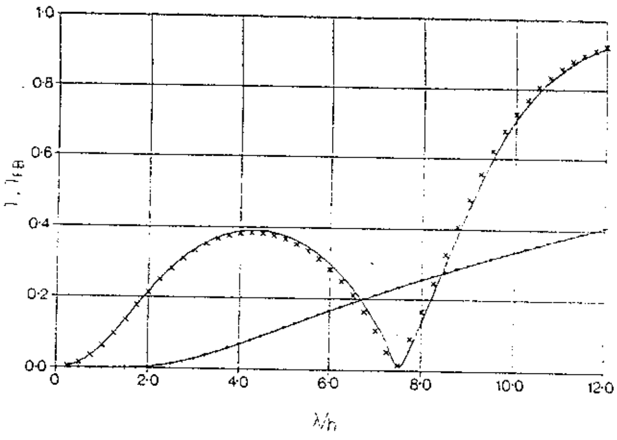

Fig. 2. Fixed and free body transmission coefficient, analytical and numerical solution comparison. (Fixed: analytical ——; numerical + + +. Free: analytical ——; numerical × × ×.) $2B/h = 2$, $d/h = 0{\cdot}7$, $\overline{GM}/h = 0{\cdot}1$, $\overline{KG}/h = 0{\cdot}72$.

In the approximated method, instead of the general solution in region II we use an approximation which is a particular solution that satisfies eqns (2), (4) and (5), plus an unknown constant for symmetric modes and a uniform horizontal velocity potential for antisymmetric modes. Instead of pressure continuity, we satisfy only an integral momentum condition, which we can solve to find explicitly the one unknown constant.

By scaling arguments the simplifying assumption was shown to fit the case of small ($S/2B$) and small (S/λ). On the other hand, for very short waves, compared to the FB draft, the flow due to the waves is small at the depths of the gap, $z < -d$, and hence for small ($S/2B$) the analytical solution is expected to fit the full linear solution for the two limiting cases — very short and very long waves. The last argument will be verified in the following section, using numerical results.

Figure 2 shows a comparison of the transmission coefficient. for a configuration that fits the assumption of small $S/2B$ ($= 0{\cdot}15$), both models are in good agreement. Figure 3 shows transmission coefficients comparison for $S/2B = 0{\cdot}36$, which is not so small; even in this case the agreement is fairly good. The calculation of the transmission coefficient requires the solution of all four sub-problems and hence obtaining correct values confirms all the stages of the solution. Figures 4–8 show a comparison of the results obtained for each sub-problem separately.

We see that all the results associated with the horizontal

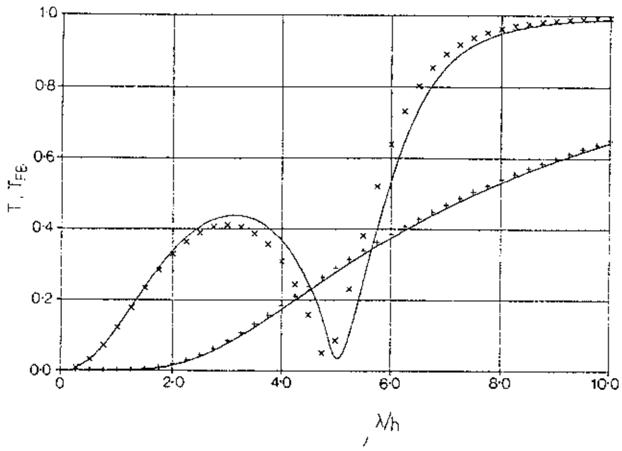

Fig. 3. Fixed and free body transmission coefficient, analytical and numerical solution comparison. (Fixed: analytical ——; numerical + + +. Free: analytical ——; numerical × × ×.) $2B/h = 1{\cdot}4$, $d/h = 0{\cdot}5$, $\overline{GM}/h = 0{\cdot}1$, $\overline{KG}/h = 0{\cdot}48$.

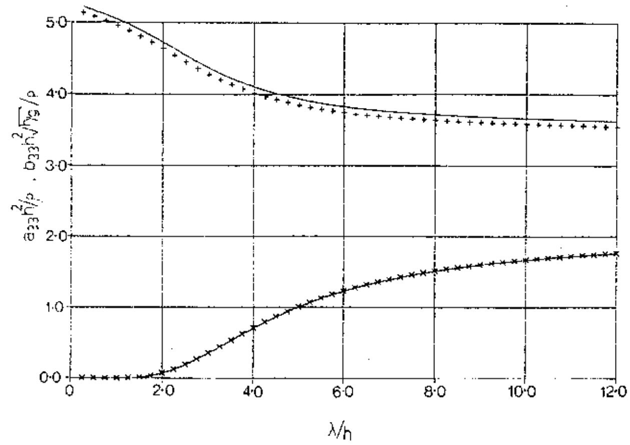

Fig. 6. Heave hydrodynamic coefficients, analytical and numerical solution comparison. (a_{33}: analytical ——; numerical + + +. b_{33}: analytical ——; numerical × × ×.) $2B/h = 2$, $d/h = 0{\cdot}7$.

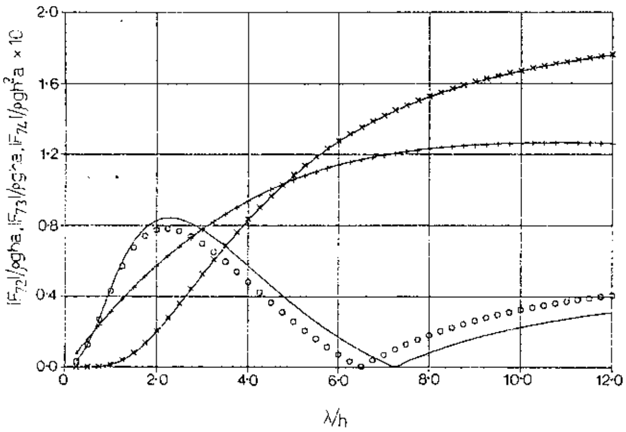

Fig. 4. Exciting forces on fixed body, analytical and numerical solution comparison. ($|F_{72}|$: analytical ——; numerical + + +. $|F_{73}|$: analytical ——; numerical × × ×. $|F_{74}|$: analytical ——; numerical OOO.) $2B/h = 2$, $d/h = 0{\cdot}7$.

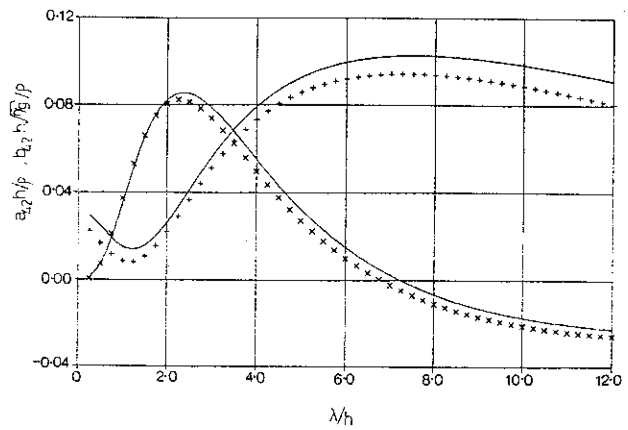

Fig. 7. Sway-roll hydrodynamic coefficients, analytical and numerical solution comparison. (a_{42}: analytical ——; numerical + + +. b_{42}: analytical ; numerical xxx.) $2B/h = 2$, $d/h = 0{\cdot}7$.

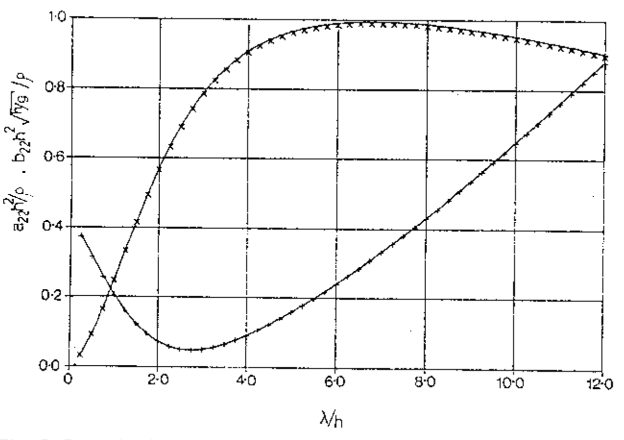

Fig. 5. Sway hydrodynamic coefficients, analytical and numerical solution comparison. (a_{22}: analytical ——; numerical + + +. b_{22}: analytical ——; numerical × × ×.) $2B/h = 2$, $d/h = 0{\cdot}7$.

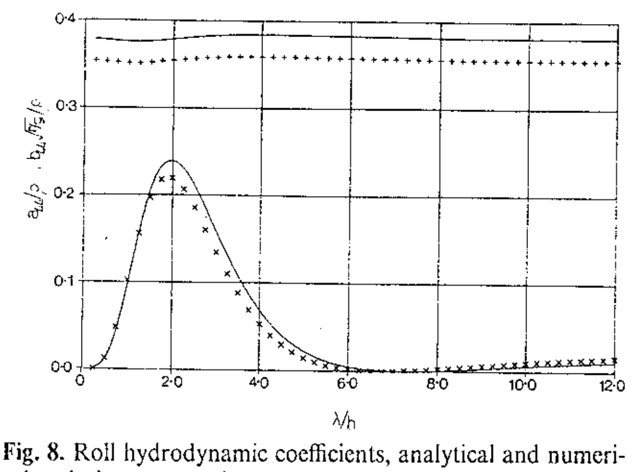

Fig. 8. Roll hydrodynamic coefficients, analytical and numerical solution comparison. (a_{44}: analytical ——; numerical + + +. b_{44}: analytical ; numerical xxx.) $2B/h = 2$, $d/h = 0{\cdot}7$.

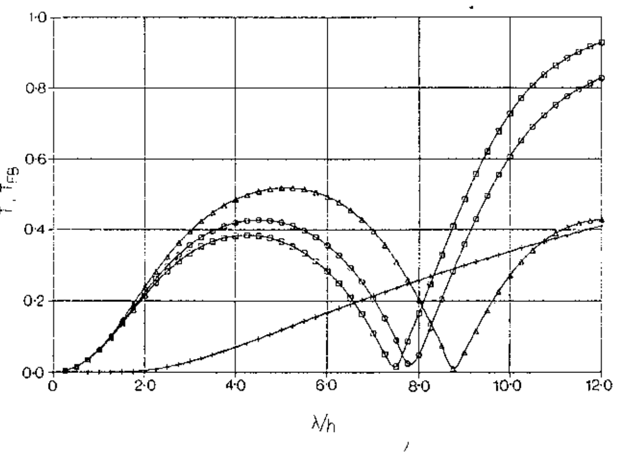

Fig. 9. Transmission coefficient, for various mooring stiffnesses. (Free □—□; mooring spring $S_2/\rho gh$ = 0·2 ○—○; $S_2/\rho gh$ = 0·5 △—△; fixed ++.) $2B/h$ = 2, d/h = 0·7, $\overline{GM}/h$ = 0·1, $\overline{KG}/h$ = 0·72.

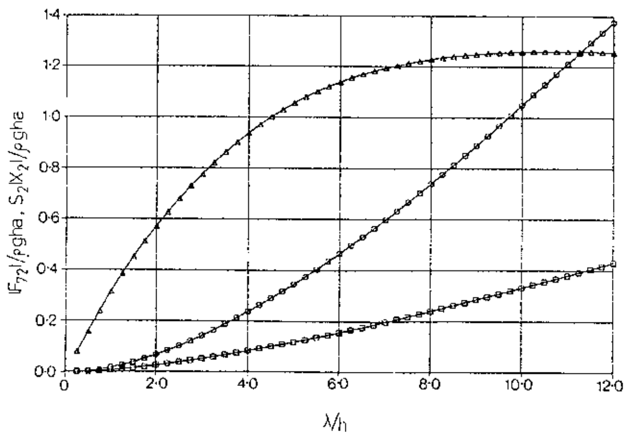

Fig. 12. Horizontal mooring force, for various mooring stiffnesses. (Mooring spring $S_2/\rho gh$ = 0·2 □—□; $S_2/\rho gh$ = 0·5 ○—○; fixed △—△.) $2B/h$ = 2, d/h = 0·7, $\overline{GM}/h$ = 0·1, $\overline{KG}/h$ = 0·72.

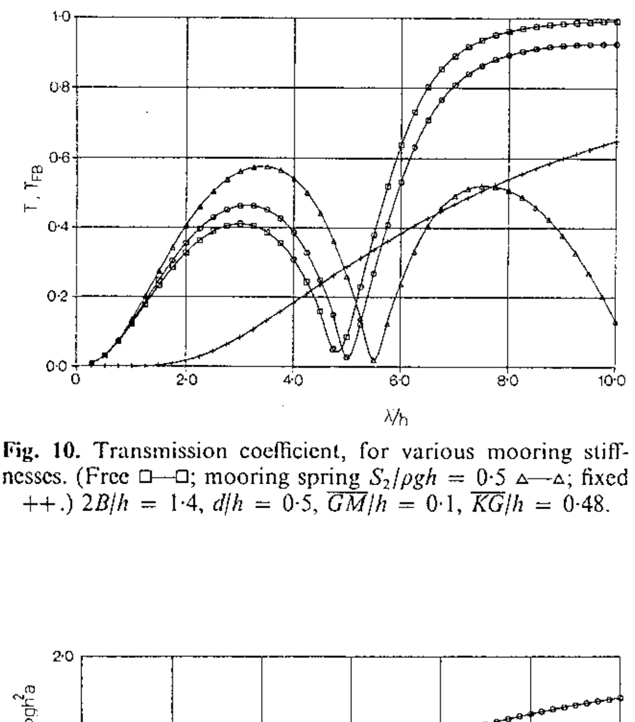

Fig. 10. Transmission coefficient, for various mooring stiffnesses. (Free □—□; mooring spring $S_2/\rho gh$ = 0·5 △—△; fixed ++.) $2B/h$ = 1·4, d/h = 0·5, $\overline{GM}/h$ = 0·1, $\overline{KG}/h$ = 0·48.

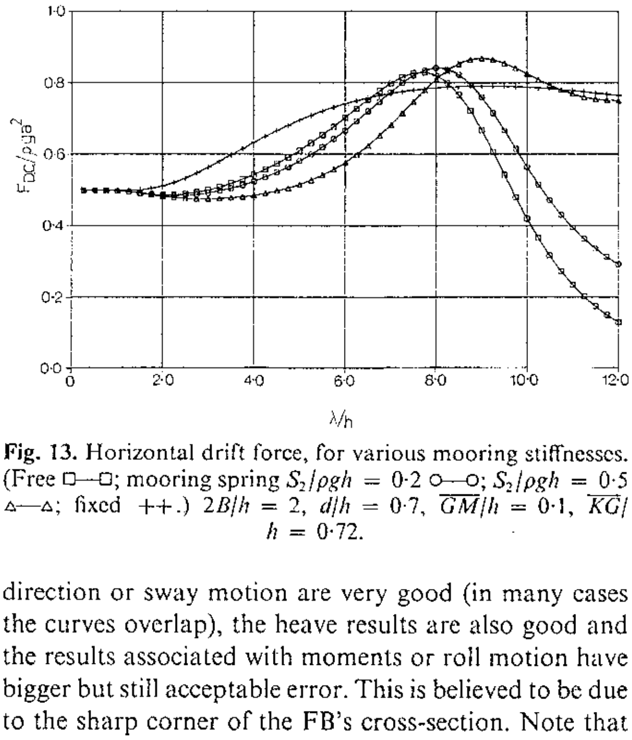

Fig. 13. Horizontal drift force, for various mooring stiffnesses. (Free □—□; mooring spring $S_2/\rho gh$ = 0·2 ○—○; $S_2/\rho gh$ = 0·5 △—△; fixed ++.) $2B/h$ = 2, d/h = 0·7, $\overline{GM}/h$ = 0·1, $\overline{KG}/h$ = 0·72.

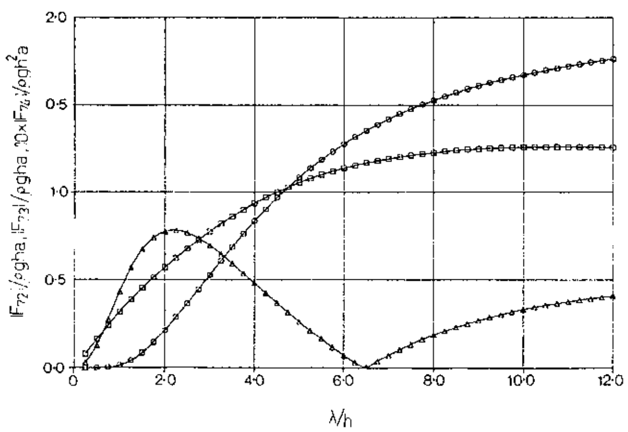

Fig. 11. Exciting forces on fixed body. ($|F_{72}|$ □—□; $|F_{73}|$ ○—○; $|F_{74}|$ △—△.) $2B/h$ = 2, d/h = 0·7.

direction or sway motion are very good (in many cases the curves overlap), the heave results are also good and the results associated with moments or roll motion have bigger but still acceptable error. This is believed to be due to the sharp corner of the FB's cross-section. Note that if the roll resonance frequency is not in the range of the incident waves frequency, the combined problem will hardly be affected by roll and hence good agreement is achieved for transmission coefficients.

In the following section we use our results to study the performance of FB's in water of intermediate depth and its dependence upon the mooring stiffness.

5 FB PERFORMANCE AT INTERMEDIATE WATER DEPTH

Figure 9 shows that in water of depth 10 m, a 20 m wide, 7-m deep freely floating structure will transmit less than 25% of the energy flux in waves of up to 90 m long.

At the same water depth a freely floating structure with 14 m breadth and 5 m draft will transmit less than 25% of the energy flux in waves not longer than 57 m (Fig. 10). The energy flux associated with long waves is almost uniform with depth and hence in order to achieve good protection we have to block a substantial part of the water depth.

The first example, shows the ability of a floating structure, with the dimensions of a typical merchant vessel, to provide a considerable protection against quite long waves. We follow with the investigation of the mooring forces required.

Figure 11 shows the exciting forces on a fixed body. Note that all the quantities are nondimensionalized by choosing the water depth h as the unit of length, the gravity acceleration g as the unit of acceleration, and the water density ρ as the unit of density.

For the case of 10-m water depth the horizontal force will be of the order of 10 ton per 1 m breakwater length per 1 m wave amplitude, which is very large.

Figure 12 shows the horizontal force that will act on mooring systems of various stiffnesses (presented by linear spring constants). The curve shows the amplitude of the force which oscillates about a mean drift force shown in Fig. 13. For the case of 10-m water depth and 1-m wave amplitude, the drift force will be of the order of 1 ton per 1-m breakwater length, which is much smaller than the oscillatory force.

From Figs 9 and 12 together we can see that a compliant mooring system ($S_2 = 0{\cdot}2$) will hardly affect the transmission coefficient as compared to a free body. A stiff mooring system ($S_2 = 0{\cdot}5$), which reduces the transmission coefficient significantly causes large forces, sometimes even larger than those on a fixed body.

Hence, it seems that a practical solution for large structures, would be a compliant mooring system which will react to the drift force only.

6 CONCLUSIONS

A simplifed model has been developed to solve analytically the two-dimensional linearized hydrodynamic problem of a pontoon type FB. As a result, all the hydrodynamic quantities of interest, given by simple analytical expressions, become accessible for routine engineering applications.

A comparison of these results with a numerical solution of the full linear problem, shows good agreement for a wide range of parameters.

Results obtained here indicate that in intermediate water depth an FB can provide good protection against waves of wavelengths ranging up to a few times the width of the structure, provided the clearance is small enough.

Our findings suggest that a flexible mooring system, which reacts to the drift force only, is a more practical solution for large structures.

Acknowledgement

The present work is part of an MSc thesis by N. Drimer, submitted to the Faculty of Civil Engineering, Technion.

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