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Remote Sensing of the Roughness of a Fractal Sea Surface

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One use of radar altimeters is to measure surface wind speeds through their effect on the roughness of the sea surface. The specular point reflection model is only appropriate for surfaces with roughness on scales which are large in relation to the radar wavelength. This may not be the case for ocean surfaces. Here we model the sea surface as a fractal on the relevant scales. This is based on Hasselmann's model for nonlinear wave action transfer. The radar cross section for nadir backscatter is derived and its dependence on the radar frequency is determined. The results are compared with the cross section obtained by the specular point model for a smoothed surface, yielding the appropriate cutoff wavenumber. Some existing measurements of the sea surface by altimeter and stereophotogrammetry are discussed, suggesting a smaller fractal dimension for short gravity waves. Dual-frequency altimeters may help determine the synoptic spectral shape.

1. INTRODUCTION AND PHYSICAL BACKGROUND

The small-scale roughness of the sea surface is an important oceanographic parameter which is observable by a number of remote-sensing devices: the radar altimeter, the scatterometer and the synthetic-aperture radar (SAR). Here we shall only discuss the altimeter, which is nadir viewing. The approach can be extended to other devices.

The sea surface roughness is observed through its effect on the radar cross section (RCS). The RCS data can be utilized to derive surface wind speeds [*Mognard and Lago*, 1979].

The traditional specular point model [*Barrick*, 1974] relates the radar cross section to the mean square slope (MSS) of the ocean surface (which, in turn, is related to the wind speed).

In this respect there are two difficulties related to the high-wavenumber end of the ocean wave spectrum. One difficulty is that the radar cross section dependence on the surface roughness based on the geometrical optics model is insensitive to the radar wavelength λ. This is only expected to be true at the limit of very short radar wavelength (compared with the smallest scale of roughness of the sea surface). Indeed, experimental data exhibit λ dependence. The second difficulty is that the decay of the spectrum at high wavenumbers is such that the MSS diverges and is very sensitive to the range of wavenumbers considered. *Stiassnie* [1988] has shown that over a range of scales comparable with the radar altimeter wavelength the sea surface may be described as a fractal surface. Fractal surfaces have no defined slope and hence no MSS.

Roughness on scales which are small compared with λ is not expected to have a significant effect on the RCS, yet it has a dominant contribution to the MSS. Therefore a naive usage of the MSS to calculate the RCS is inadequate.

Barrick and Lipa [1985] addressed this issue early on, suggesting a cutoff proportional to λ. Using the full wave theory, *Bahar et al.* [1983] have discussed the determination of an appropriate high wavenumber cutoff, k_d, to be used in the MSS evaluation. Their result for the SEASAT altimeter is $k_d = 85\ m^{-1}$ and is quite insensitive to the wind speed. Using this result and the wind speed dependence of the RCS found by *Chelton and McCabe* [1985], *Barrick and Bahar* [1986] derive a spectral law for the wavenumber spectrum, with a power of -3.86.

Glazman [1990] gives a qualitative analysis of the dependence of the spectral power law dependence on a nondimensional fetch parameter, i.e., the degree of development of the sea. He models the spectral density by a power law and introduces an exponential high-wavenumber decay at an intrinsic microscale of 0.4 m which corresponds to k_d=2.5 m^{-1}, and a rapid low-wavenumber cutoff. He then determines the range over which the geometrical optics result yields a good approximation.

Simple analytic forms for the RCS are obtainable in two distinct ways. One approach is to introduce a cutoff scale that is related to λ [*Glazman*, 1986]. This is equivalent to taking a smoothed surface. Alternatively, the smaller scales may be taken into account through theories for "diffractal echoes" - waves scattered from fractal surfaces [*Berry*, 1979; *Berry and Blackwell*, 1981].

In this paper these two approaches are applied to the problem of measuring the sea surface roughness by near-nadir radar observations. The results of the two approaches are compared, and the implications for the dependence of the RCS on the wave spectrum for different radar frequencies is discussed. The results can be utilized in dual-frequency altimetry.

Knowledge of the spatial structure of the sea surface has been increasing but is still lacking, especially at the small scales. *Banner et al.* [1989] discuss some existing measurements. In their study they have used stereophotogrammetry to determine the wavenumber spectrum of the sea for wavelengths in the range 0.2-1.6 m. The spectra they obtained were nearly isotropic. They also determined the power law dependence, which will be discussed in section 6, and discussed possible theories for the equilibrium between nonlinear wave interaction and dissipation mechanisms (cf. also *Phillips* [1985]).

The analysis for scattering from a rough surface (including the fractal case) is most readily applied to isotropic or to corrugated surfaces. Banner et al. note that waves shorter and longer than those studied by them are often nearly unidirectional. Such waves can also be treated by a simple theory. In order to simplify the analysis we focus here on the isotropic case.

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It is generally accepted that at small scales the spectrum has a faster falloff rate than in the equilibrium range. This is due to the larger role played by surface tension and by dissipation and may also be related to coupling with the airflow. It is the small scales that dominate the RCS. Rather than truncate the spectrum, we model it as a fractal on the relevant scales. This allows us to evaluate in full the Kirchhoff approximation (the specular point result is a further approximation, using the method of steepest descent, which is valid only for smooth surfaces).

We note that in this modeling of the sea surface as a fractal, the basic premise of the Kirchhoff approximation, of small enough curvature, need not be violated, since at the smallest scales the surface is, of course, differentiable (on capillary wave scale it is a subfractal, cf. *Stiassnie et al.*, [1991]). See *Glazman* [1990] for a discussion of the curvature criterion . The 'fractal' model of the sea surface is limited to a finite range of scales. Geometrically, the sea surface is smooth, and its waves propagate in the familiar way.

In section 2 we calculate the radar cross section for a smoothed sea surface. Section 3 is a discussion of the fractal nature of the sea surface; in section 4 the radar cross section of a fractal surface is obtained. In section 5 the smooth and fractal results are compared and the self-affinity of the surface is used to derive a consistent cutoff criterion. Section 6 is a discussion of the implications of the results for the measurement of the roughness of gravity waves and the derivation of surface winds. In the conclusion the application of dual-frequency altimetry is discussed.

2. THE ROUGHNESS OF A DIFFERENTIABLE SURFACE

Let the random sea surface be given by the following stochastic model [*Pierson*, 1955]:

$$\eta(x,y,t)=\int_0^{\infty}\int_{-\pi}^{\pi}\cos[k(x\cos\theta+y\sin\theta)-\sqrt{gk}\ t+\varepsilon(k,\theta)]\sqrt{\psi(k,\theta)kdkd\theta} \quad (1)$$

η is the free surface elevation, (x,y) are the horizontal coordinates and t is the time. In this model the ocean surface is composed of an infinite number of waves each with its own wavenumber $k = (k\cos\theta, k\sin\theta)$, frequency $\omega = \sqrt{gk}$, and a random phase shift ε, uniformly distributed over $[-\pi, \pi]$.

We consider an isotropic wavenumber spectrum ψ of the form

$$\psi = s_0 k^{-\alpha} \qquad k_0 < k \quad (2)$$

where S_0 is the spectral constant. This is a common spectral shape for the high-wavenumber end of the spectrum, which determines the radar cross section.
For gravity waves

$$\alpha < 4$$

for which the MSS is not defined. If we smooth the surface by truncating the spectrum for

$$k > k_d >> k_0$$

we get the mean slope β^2:

$$\beta^2 = \int_{k_0}^{k_d}\int_{-\pi}^{\pi} k^3\psi dk d\theta = 2\pi\frac{s_0}{4-\alpha}(k_d^{4-\alpha}-k_0^{4-\alpha}) \cong 2\pi\frac{s_0}{4-\alpha}k_d^{4-\alpha} \quad (3)$$

The specular point model gives the following expression for the RCS

$$\sigma_s = 1/\beta^2 \quad (4)$$

[*Barrick*, 1974]. So that

$$\sigma_s = \frac{4-\alpha}{2\pi s_0}\ k_d^{-(4-\alpha)} \quad (5)$$

3. A FRACTAL MODEL OF THE SEA SURFACE

Stiassnie [1988] has shown that the spectrum of isotropic gravity waves in equilibrium due to quartet nonlinear interactions corresponds to a sea surface of fractal dimension 21/4 or 21/3 . We briefly explain the principal ideas.

The wave action transfer equation is given by *Hasselmann* [1962] as

$$\frac{\partial N}{\partial t} = 64\pi^5 \iiint \{N(\underline{k}_2)N(\underline{k}_3)[N(\underline{k})+N(\underline{k}_1)]-N(\underline{k})N(\underline{k}_1)[N(\underline{k}_2)+N(\underline{k}_3)]\}$$

$$\bullet[(T(\underline{k},\underline{k}_1,\underline{k}_2,\underline{k}_3)]^2\delta(\underline{k}+\underline{k}_1-\underline{k}_2-\underline{k}_3)\delta(\omega+\omega_1-\omega_2-\omega_3)d\underline{k}_1 d\underline{k}_2 d\underline{k}_3 \quad (6)$$

where $\psi(\underline{k})$ is related to the wave action spectral density $N(\underline{k})$ by

$$\psi(\underline{k}) = [N(\underline{k})+N(-\underline{k})]\frac{k}{\omega} \quad (7)$$

The kernel T^2 is the square of the kernel in the Zakharov equation [see *Stiassnie and Shemer* , 1984].
Consider an isotropic regime

$$N(\underline{k}) = N(k)$$

Zakharov and Zaslavskii [1982] have obtained two stationary isotropic solutions for equation (6):

$$N(k) \propto k^{-\tilde{\alpha}} \qquad \tilde{\alpha} = 23/6,\ 4 \quad (8)$$

Equations (7) and (8) and the dispersion relation for gravity waves yield

$$\psi = s_0 k^{-\alpha} \qquad \alpha = 10/3,\ \ 7/2 \quad (9)$$

Sections of the surface are obtained after substituting these spectra in equation (1):

$$\eta(r) = \eta_* \int_0^{\infty}\cos(kr+\varepsilon(k))\sqrt{k^{-\alpha}kdk} \quad (10)$$

These surface sections are fractals of dimensions

$$D_1 = \frac{6-\alpha}{2} = 4/3,\ 5/4 \tag{11}$$

[cf. *Stiassnie et al.*, 1991, section 2]. The fractal dimension of the sea surface itself is thus

$$D = D_1 + 1 = \frac{8-\alpha}{2} = 2\frac{1}{3},\ 2\frac{1}{4} \tag{12}$$

The value $D = 2^{1/4}$ which corresponds to $\alpha = 3^{1/2}$ is in agreement with the results of several field and laboratory investigations presented by *Phillips* [1985] over the range of scales from 15 m to about 0.1 m.

4. SCATTERING BY A FRACTAL SEA SURFACE

For smooth diffracting objects, geometrical optics applies. Roughness on scales much smaller than the radar wavelength λ does not affect the scattering. When there is no roughness on scales near λ, the surface may be treated as smooth. Otherwise, different tools are required.

If the structure of the surface is self-affine over a range of scales which are comparable with λ, the theory of diffractals [*Berry*, 1979, *Berry and Blackwell*, 1981] applies.

For a fractal surface the mean square slope is undefined. We will see that the diffracted field is written in terms of the mean square increment

$$\Delta(\underline{r}) \equiv \langle(\eta(\underline{x} + \underline{r}) - \eta(\underline{x}))^2\rangle \tag{13}$$

where angle brackets stand for ensemble average. For fractal surfaces of dimension D, Δ has a cusp at r=0:

$$\Delta(\underline{r}) \underset{\underline{r}\to 0}{\longrightarrow} L^{2D-4} r^{6-2D} \qquad 2<D<3 \tag{14}$$

L, the topothesy, is a characteristic length scale of the fractal. L is the distance r over which chords joining points on the surface have an rms slope of one rad. L is related to s_0 by

$$L^{4-\alpha} = \frac{4s_0}{\alpha-2}\Gamma(3-\alpha)\sin(\frac{\pi}{2}(3-\alpha))\,B(\frac{1}{2},\frac{\alpha-1}{2}) \tag{15}$$

as shown in the appendix.

For a smooth surface D=2,

$$\Delta(\underline{r}) \underset{\underline{r}\to 0}{\longrightarrow} \beta^2 r^2 \tag{16}$$

There is no topothesy, and β^2 is the MSS (equation 3).

Berry and Blackwell [1981] applied the Kirchhoff approximation to the problem of backscatter at nadir incidence. This approximation neglects shadowing and multiple scattering. This is a good assumption when making the 'Fresnel' or 'paraxial' approximation, which requires that the main incident and scattered contributions make small angles with the vertical. These conditions are definitely satisfied in satellite altimetry.

The Kirchhoff approximation gives the RCS as

$$\sigma = \frac{2k_\lambda^2}{\pi}\iint d\underline{r}\, e^{-2k_\lambda^2\Delta(\underline{r})} = 4k_\lambda^2\int r\, e^{-2k_\lambda^2\Delta(\underline{r})}dr \tag{17}$$

where k_λ is the radar wavenumber.

For radar scatter from the ocean surface, $k_\lambda^2\Delta(r)$ is zero at r =0 and is finite but large at large r. The value of σ is dominated by values of r for which

$$k_\lambda^2\Delta(r) = 0(1)$$

This is because the contribution of very large scales is exponentially small. *Yordanov and Stoyanov* [1989] have shown that for a fractal surface, a small wavenumber cutoff has a very small effect on the RCS. The details of the falloff of $\Delta(r)$ to zero at small r, too, hardly affect the value of σ.This can be seen graphically: In Figure 1*a* two structure functions are shown. One for a fractal surface and the other quadratic. The corresponding integrands on the right-hand side of Eq. (17) are plotted in Figure 1*b*. The area under the curve represents the RCS. We see that for a fractal structure extending to scales of the order of 0.1 m and a topothesy much smaller than λ (i.e. s_0 not too large), the two integrands are very close over the small scales. The contribution from very large r is small and can be disregarded, since in practice, the integration range is limited by the altimeter beam form and the curvature of the Earth and the radar wave front, which have been neglected due to the short pulse of the altimeter [cf. *Barrick and Lipa*, 1985].

Substituting the mean square increment from Eq. (14) into Eq. (17) gives the RCS in closed form:

$$\sigma_F = \frac{2k_\lambda^2}{\pi}\iint d\underline{r}\, e^{-2k_\lambda^2 L^{2D-4} r^{6-2D}} = 4k_\lambda^2\int r dr\, e^{-2k_\lambda^2 L^{2D-4} r^{6-2D}}$$
$$= \frac{1}{3-D}(\sqrt{2}\,k_\lambda L)^{-\frac{2D-4}{3-D}}\Gamma(\frac{1}{3-D}) \tag{18}$$

In the limit $D \to 2$, L^{2D-4} is replaced by β^2, and σ_F reduces to $\sigma_s = 1/\beta^2$. When the surface is fractal, σ_F is a function of k_λ.

5. AFFINITY ANALYSIS AND SPECTRAL SPLITTING

In this section we address the question of what should be the criterion for selecting a cutoff wavenumber if one wishes to approximate the fractal surface by a smoothed one. It is emphasized that this smoothing is not necessary in view of the solution for the RCS of a fractal surface presented in

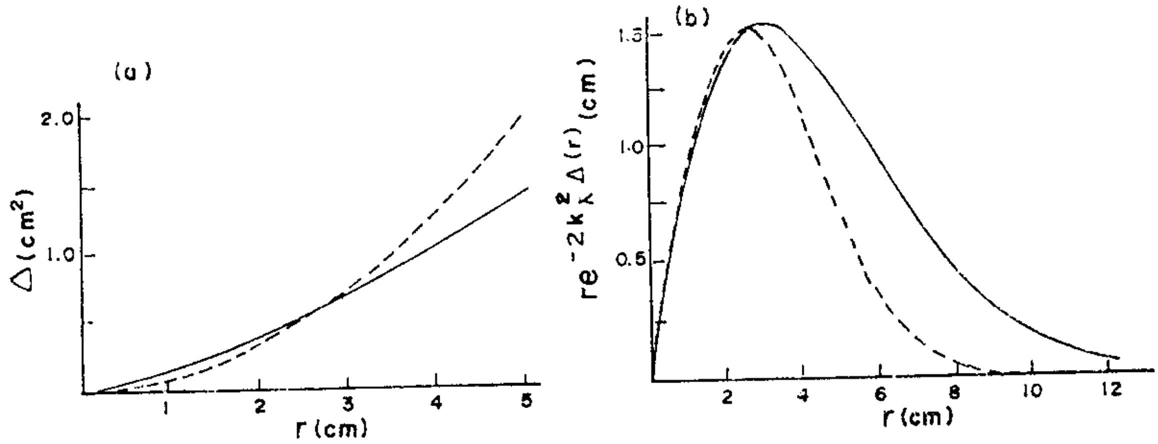

Fig. 1. (*a*) The mean square increment versus r. Solid line: fractal regime D = 2.25, $s_0 = 5\text{x}10^{-3}$ m$^{0.5}$). Dashed line: quadratic form. (*b*) The integrand on the right hand side of equation (17). Solid line: using fractal shape. Dashe line: using the quadratic function for all scales (λ = 6 cm).

section 4. However, physical insight may be gained by refering the results to the smoothed surface results.

The simplest cutoff, which has been in use, is one proportional to k_λ:

$$k_d \propto k_\lambda \tag{19}$$

Using equation (5) gives

$$\sigma_s \propto s_0^{-1} k_\lambda^{-(4-\alpha)} \tag{20}$$

For σ_F we found (equations (12) and (18))

$$\sigma_F \propto (Lk_\lambda)^{-\frac{2D-4}{3-D}} = (Lk_\lambda)^{-\frac{8-2\alpha}{\alpha-2}} \propto s_0^{-2/(\alpha-2)} k_\lambda^{-(8-2\alpha)/(\alpha-2)} \tag{21}$$

from that of σ_F except for $\alpha = 4$ (D=2). This is because a fractal surface is self-affine rather than self-similar: Different factors of stretching apply in the horizontal and in the vertical directions. To find the correct dependence of the cutoff wavenumber k_d on k_λ and s_0, we note that in the Kirchhoff approximation, the determining factor is the phase dependence in the ensemble average, (see equation (4.8) of *Berry and Blackwell* [1981]). This is of the form

$$k_\lambda(\eta(r_1)-\eta(r_2)) \tag{22}$$

To find the dependence of k_d on k_λ, we fix s_0 (no vertical stretching of the sea surface). Now replace k_λ by

$$k'_\lambda = \gamma^{-1} k_\lambda \tag{23}$$

To maintain the same phase dependence, $\eta(r_1)-\eta(r_2)$ should be increased γ-fold. This can be achieved by shifting the cutoff k_d. Self-affinity gives

$$\Delta\eta(\gamma^{1/H} r) = \gamma\Delta\eta(r) \tag{24}$$

where $\Delta\eta(r)$ is the distribution of differences in η over a distance r and

$$H = \frac{\alpha-2}{2} \tag{25}$$

[cf. *Stiassnie et al.*, 1991, section 2].This means that we should replace r by $\gamma^{1/H}$r. Thus if the cutoff appropriate for k_λ is k_d, the cutoff for k'_λ should be

$$k'_d = \gamma^{-1/H} k_d = \gamma^{-\frac{2}{\alpha-2}} k_d \tag{26}$$

To find the dependence of k_d on s_0, we fix k_λ. Now replace s_0 and η by

$$s'_0 = \gamma^{-1} s_0, \quad \eta'(r_1) = \gamma^{-1/2}\eta(r_1) \tag{27}$$

In order to maintain

$$k_\lambda \Delta\eta'(r') = k_\lambda \Delta\eta(r) \tag{28}$$

we must use the mapping

$$r' = \gamma^{1/2H} r \tag{29}$$

Hence the appropriate cutoff is modified to

$$k'_d = \gamma^{-\frac{1}{\alpha-2}} k_d \tag{30}$$

The combined dependence of k_d on s_0 and k_λ is

$$k_d = c(\alpha)\, s_0^{\frac{1}{\alpha-2}} k_\lambda^{\frac{2}{\alpha-2}} \qquad (31)$$

where $c(\alpha)$ is a dimensionless coefficient.

When calculating the RCS for the smoothed surface with a cutoff determined in this way we find

$$\sigma_s \propto s_0^{-\frac{2}{\alpha-2}} k_\lambda^{-\frac{8-2\alpha}{\alpha-2}} \qquad (32)$$

this is the same form found for σ_F (equation (21)).

We may determine $c(\alpha)$ in (31) by comparing σ_s from equation (5) with σ_F from equation (18):

$$\sigma_s = \frac{4-\alpha}{2\pi s_0} k_d^{-(4-\alpha)} = \frac{(4-\alpha)c(\alpha)^{-(4-\alpha)}}{2\pi} s_0^{-\frac{2}{\alpha-2}} k_\lambda^{-\frac{8-2\alpha}{\alpha-2}} \qquad (33)$$

$$\sigma_F = \frac{1}{3-D}(\sqrt{2}\, k_\lambda L)^{-\frac{2D-4}{3-D}} \Gamma\left(\frac{1}{3-D}\right) \equiv q(\alpha) s_0^{-\frac{2}{\alpha-2}} k_\lambda^{-\frac{8-2\alpha}{\alpha-2}} \qquad (34)$$

where

$$q(\alpha) \equiv \frac{2}{\alpha-2}\left(\frac{2}{\alpha-2}\right)^{-\frac{8-\alpha}{2}} \left(\sin\frac{\pi}{2}(3-\alpha) B\left(\frac{1}{2}, \frac{\alpha-1}{2}\right)\Gamma(3-\alpha)\right)^{-\frac{2}{\alpha-2}} \Gamma\left(\frac{2}{\alpha-2}\right)$$

$q(\alpha)$ has the value 3.5×10^{-2} for $\alpha = 3.5$. In order to make $\sigma_s = \sigma_F$ for a particular case, say $\lambda = 0.03$ m and $s_0 = 10^{-3}\ m^{0.5}$ (which corresponds to a wind speed of about 9 m/s at 10 m above sea level), we need to set $c(3.5) = 4.99$. This yields a cutoff wavelength of 0.11 m.

Bahar et al. (1983) have studied the question of spectral splitting: the decoupling between large-scale and small-scale roughness. The criterion they used was

$$4k_\lambda^2 \langle\eta_R^2\rangle = \beta_0 = \text{const.} \qquad (35)$$

where

$$\langle\eta_R^2\rangle = \int_{k>k_d} \psi d\underline{k} \qquad (36)$$

is the rms of the remainder surface elevation with scales smaller than $2\pi/k_d$. This corresponds to the requirement of neglecting small roughness. Bahar et al. have not studied a power law spectrum.

$$\frac{\beta_0}{4k_\lambda^2} = \langle\eta_R{}^2\rangle = \frac{2\pi s_0}{\alpha-2} k_d^{-(\alpha-2)} \qquad (37)$$

so

$$k_d = \left(\frac{8\pi}{\beta_0(\alpha-1)}\right)^{\frac{1}{\alpha-2}} s_0^{\frac{1}{\alpha-2}} k_\lambda^{\frac{2}{\alpha-2}} \qquad (38)$$

This is exactly the form obtained in equation (31).

6. REMOTE SENSING OF THE SEA SURFACE ROUGHNESS

Finally, we wish to examine the relation of the radar cross section to the wind speed through the small-scale roughness. Much work has been done on relating measured cross sections with wind speed since the early work of *Cox and Munk* [1954] who studied photographs of the sun glitter. The use of remote sensing to derive the wind speed from the small-scale roughness relies on the model employed for relating the RCS to the surface roughness. A form that is commonly used is

$$\sigma_F = G u^\mu \qquad (39)$$

where G and μ are constants and u is the wind speed at the given elevation.

From *Phillips* [1985] we get the following relation for the spectral constant:

$$s_0 \propto u_* / g^{1/2} \qquad (40)$$

where u_*, the friction velocity, is nearly proportional to the wind speed u. This would imply

$$\sigma_F \propto s_0^\mu \qquad (41)$$

We have found (equations (12) and (21))

$$\sigma_F \propto s_0^{-\frac{2}{\alpha-2}} = s_0^{-\frac{2}{6-2D}} \qquad (42)$$

Equation (40) which is specific to $\alpha = 3.5$, may be extended using the affinity analysis (as well as through dimensional arguments) with the result that s_0 should be of the form

$$s_0 = q_1(\alpha)\left(\frac{u_*^2}{g}\right)^{4-\alpha} \qquad (43)$$

where $q_1(\alpha)$ is a dimensionless function (which for $\alpha = 3.5$ corresponds to $1/4\pi$ of the coefficient denoted α by *Phillips* [1985]. For $\alpha = 3.5$ equation (43) reduces to the form (40).

Combining (34) and (43) we get

$$\sigma_F = q_2(\alpha)\left(\frac{u_*^2}{g}k_\lambda\right)^{-\frac{8-2\alpha}{\alpha-2}} \tag{44}$$

where

$$q_2(\alpha) = q(\alpha)q_1(\alpha)^{\frac{2}{\alpha-2}} \tag{45}$$

In particular, we have

$$\sigma_F \propto u_*^{-4/3}\,k_\lambda^{-2/3}, \qquad \text{for} \quad \alpha = 3\frac{1}{2} \tag{46a}$$

$$\sigma_F \propto u_*^{-2}\,k_\lambda^{-1}, \qquad \text{for}\ \alpha = 3\frac{1}{3} \tag{46b}$$

Using the expression (44) the friction velocity u_* may be determined from measurements of σ_F. This requires that we assume a given value for α, say 3.5. For this case the data from *Phillips* [1985] correspond to $q_1 = 0.01$ and $q_2 = 16$. Using a dual-frequency altimeter, measuring the RCS for two radars with different frequencies should allow the estimation of α as well as u_*. This is the case when both frequencies are in the range in which scattering is dominated by gravity waves in the equilibrium range.

We stress that caution is required in determining the parts of the spectrum that affect the radar scatter for a given radar frequency:

1. The scatter process is related to the vertical scales of the surface (which are smaller than the horizontal scales). This is manifested by the role played by the structure function Δ (or the topothesy, in the fractal regime). If the surface is rougher, smaller horizontal scales become important.

2. The value of the structure function itself, for a given horizontal separation r, is determined by the whole spectrum, notably by waves with wavelength 2r.

These two considerations imply that the relevant wavelengths of the surface waves need not coincide with the radar wavelength.

Short ocean waves (from perhaps 3 cm wavelength) are affected by surface tension and tend to be directional. This regime is less understood than the gravity wave equilibrium range. The contribution from shortwaves becomes more important for higher radar frequency and higher sea states.

Another source of uncertainty stems from the effect of the degree of development of the sea, which is related to the fetch [cf. *Glazman*, 1990]. Partially developed seas have a faster falloff rate and hence a smaller fractal dimension. Averaging over different conditions should result in some average values. As an illustration, let us examine available altimeter measurements in the K_u band. These are affected by shorter ocean waves which have a faster falloff rate (larger α). By (44) this would imply a weaker wind dependence. Indeed, wind dependence was found to be rather weak [*Brown*, 1979]. The RCS dependence on u found by *Chelton and McCabe* [1985] is

$$\sigma \propto u^{-0.468}$$

From (44) this corresponds to $\alpha = 3.79$:

$$\sigma_F \propto u_*^{-0.468}\,k_\lambda^{-0.234} \qquad \text{for}\ \alpha = 3.79 \tag{46c}$$

This is consistent with the results of *Banner et al.* [1989], who found a spectral dependence on the wind speed in the form of (43). They obtained values corresponding to $\alpha = 3.91 \pm 0.09$ with confidence limits of 95%. We recall that *Barrick and Bahar* [1986] have obtained $\alpha = 3.86$. The value of $\alpha \cong 3.8$ appearing in (46*c*) corresponds to a fractal dimension of 2.1 in the range of decimeter gravity waves.

7. Conclusion

We have examined the scattering of radar waves from isotropic rough sea surfaces with power law spectra. The 'specular point' and the 'diffractal' models were compared. Using affinity analysis, it was shown that some features of the fractal model may be simulated by artificially introducing a high-wavenumber spectral cutoff to the specular point model.

For K_u- band altimeters, it was shown that both remote sensing results [*Chelton and McCabe*, 1985] and stereophotogrammetry [*Banner et al.*, 1989] indicate a possible fractal dimension of 2.1. More measurements are required to determine the structure of the sea surface on centimeter and decimeter scales. Remote-sensing techniques hold great promise in obtaining such data. If the fractal dimension of the surface is assumed to be known, a single frequency measurement of the RCS can yield the spectral constant and subsequently the wind speed. The simplified model obtained here (or further refinements) and the use of dual-frequency altimeters are expected to yield the fractal dimension as well. Future applications can include underwater acoustic measurement of the wind speed, as well as off-nadir scattering.

Appendix: The Topothesy of the Sea Surface

From Figure 3 of *Phillips* [1985], which gives a dimensionless plot of field and laboratory measurements of the frequency spectrum in the equilibrium range (α=3.5), we calculated the constant s_0 in (9) and found that for u^*=0.37 m/s, $s_0 \cong 10^{-3}\,m^{0.5}$. The mean square increment, defined in (13), can be calculated from the spectrum $\psi(k)$ via

$$\Delta(\underline{r}) = 2\iint d\underline{k}\,\psi(\underline{k})(1-\cos(\underline{k}\bullet\underline{r})) \tag{A1}$$

Substituting (9) into (A1), integrating by parts, and then using equations 3.761(4) and 8.380(2) of *Gradshteyn and Ryzhik* [1980] yield

$$\Delta(\underline{r}) = \frac{4s_0}{\alpha-2}\Gamma(3-\alpha)\sin\left(\frac{\pi}{2}(3-\alpha)\right)B\left(\frac{1}{2},\frac{\alpha-1}{2}\right)r^{\alpha-2} \tag{A2}$$

where Γ, B are the Gamma and Beta functions, respectively. From (14), (12) and (A2) the topothesy L of the sea surface $\eta(\underline{x})$ is given by

$$L = \{\frac{4s_o}{\alpha-2}\Gamma(3-\alpha)\sin(\frac{\pi}{2}(3-\alpha))B(\frac{1}{2}, \frac{\alpha-1}{2})\}^{\frac{1}{4-\alpha}} \quad (A3)$$

For $\alpha = 3.5$ the last equation gives $L = 1.3 \times 10^{-4}$ m. This seemingly small value is due to the definition of the topothesy as the distance over which joining chords have rms slope of 1 rad.

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