# Scattering and dissipation of surface waves by a bi-plate structure

MICHAEL STIASSNIE

*Department of Civil Engineering, Technion, Haifa 32000, Israel*

IRINA BOGUSLAVSKY and EHUD NAHEER

*Coastal and Marine Engineering Research Institute, Technion, Haifa 32000, Israel*

Methods for the calculation of scattering and dissipation of surface waves by a bi-plate structure are provided and compared with wave-flume experiments. The theoretical results are in good agreement with the measurements.

## 1. INTRODUCTION

The considered problem is that of two rigidly-held parallel vertical plates attacked perpendicularly by long-crested surface waves. The specific conditions consist of a two-dimensional configuration and infinitely deep water (see Fig. 1). Approximate analytical solutions to this problem, within the framework of irrotational and linear wave theory, can be found in Evans and Morris,[1] Srokosz and Evans,[2] and Stiassnie, Agnon and Naheer.[3] The solutions in the last two references are based on the wide-spacing approximation method. The method, first used by Ohkusu,[4] is based on the assumption that the plates are spaced far enough for the local wave field in the vicinity of one plate not to influence the other plate. The only interaction between the plates is due to the propagating-wave terms which occur in the scattering problem for a single plate. Mathematically, this approximation is equivalent to the assumption that the wave length is small compared with the distance between the plates; but it has been shown by Ohkusu[4] that the method is valid over a much wider range of the ratio of wave length to plate spacing. The first of the two main goals of the present work is to provide an experimental test for the range of applicability of the wide-spacing approximation.

It is a well-known fact that sharp edges, like those at the bottom of the plates, cause boundary layer separation and tend to shed vortices (see Fig. 7 in Knott and Mackley[5]).

These vortices are shed in pairs, one pair from each edge per wave period, and move downwards and away from the edges. The kinetic energy of the vortices is eventually converted into heat by viscous dissipation. Recently, Stiassnie, Naheer and Boguslavsky,[6] have provided a simple model to estimate the amount of energy taken away from the waves by the vortex generation process. This model, which was derived for a single plate configuration, was found to agree well with experimental results. In the case of a single plate the fraction of energy lost to the vortices is less than 15%. For a two-plate configuration this fraction can reach as high as 50%. This happens when nearly standing waves, of relatively high amplitude, are trapped between the two plates.

The second goal of this work is to examine how well our single plate vortex dissipation model works for the bi-plate configuration.

The mathematical solutions are briefly summarised in the following section; the experimental results and their comparison with both theories are given in section 3; with some concluding remarks in section 4.

## 2. THEORY

*Scattering*

Referring to Fig. 1, and denoting the incident wave by $\eta_I$, we write:

$$\eta_I = a_{i_0} \operatorname{Re}\{\exp i(\sigma t - kx)\} \tag{1}$$

where $a_{i_0}$ is the wave amplitude; $\sigma$ and $k$ are respectively the frequency and wavenumber related by the linear dispersion relation $\sigma^2 = gk$, and $t$ is the time. $i$ is the imaginary unit and Re {*} stands for the real part of *.

From section 5.3 and equation (6.4) in Srokosz and Evans,[2] one finds that the transmitted ($\eta_T$) and reflected ($\eta_R$) waves are given respectively by:

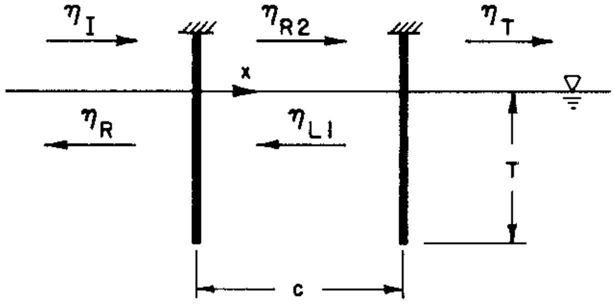


*Figure 1. Definition sketch for waves approaching a bi-plate structure*

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$$\eta_T = a_{i_0} \operatorname{Re}\{T_c \exp i(\sigma t - kx)\}; \quad T_c = E^2(1-r)^2/(E^2-r^2) \tag{2}$$

$$\eta_R = a_{i_0} \operatorname{Re}\{R_c \exp i(\sigma t - kx)\}; \quad R_c = r + r(1-r)^2/(E^2-r^2) \tag{3}$$

$T_c$ and $R_c$ are the complex transmission and reflection coefficients; $E = \exp i(kc)$ where $c$ is the distance between the plates; $r$ is the reflection coefficient for a single plate, given by:

$$r = \pi I_1(kT)/(\pi I_1(kT) - iK_1(kT)) \tag{4}$$

where $K_1$ and $I_1$ are modified Bessel functions and $T$ is the draught of the plates. The waves propagating to the right ($R_2$) and to the left ($L_1$) in the space between the two plates are given by:

$$\eta_{R_2} = a_{i_0} \operatorname{Re}\{E R_2 \exp i(\sigma t - kx)\}; \quad R_2 = E(1-r)/(E^2-r^2) \tag{5}$$

$$\eta_{L_1} = a_{i_0} \operatorname{Re}\{L_1 \exp i(\sigma t - kx)\}; \quad L_1 = r(1-r)/(E^2-r^2) \tag{6}$$

These results were obtained by applying the wide-spacing approximation and are a particular case of the general multi-plate solution given in Stiassnie, Agnon and Naheer.[3]

*Dissipation*

For a single plate attacked by waves from one side only it was found[6] that the energy flux (per unit width) from the wave field into the vortex motion is:

$$e_{v_0} = \frac{0.54\rho g^{3/2} k^{1/6}}{[kT(K_1^2 + \pi^2 I_1^2)]^{4/3}} (a_{i_0})^{8/3} \tag{7}$$

where $\rho$ is the fluid density.

For plate No. 1 in Fig. 1, which is attacked by waves from both sides ($\eta_I$ from the left and $\eta_{L_1}$ from the right) one can show that the dissipation is given by:

$$e_{v_1} = |1 - L_1|^{8/3} \cdot e_{v_0} \tag{8}$$

Plate No. 2 is attacked by $\eta_{R_2}$ only and its vortex dissipation is

$$e_{v_2} = |R_2|^{8/3} \cdot e_{v_0} \tag{9}$$

The total vortex dissipation $e_v$ is given by the sum of $e_{v_1}$ and $e_{v_2}$.

## 3. EXPERIMENTAL VERIFICATION

*The model*

Tests were conducted in a 27 m long, 60 cm wide and 1.30 m deep wave channel (see Fig. 2). The channel is equipped with a piston-type wave generator, the motion of which is controlled by an external (electronic) signal. For the present investigation, sinusoidal signals were used in order to generate monochromatic waves. An absorbing beach made of rubberised hair and having a slope of 1 : 3 was installed at the downstream end of the channel.

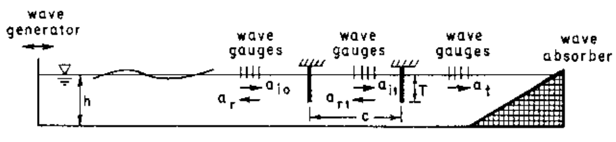


*Figure 2. The experimental set-up*

Two plates which spanned from wall to wall across the channel were rigidly fixed to the channel concrete walls. The gaps between the plates and the walls were sealed. The upstream plate was at a distance of 13 m from the wave generator.

Tests were performed for two values of spacing to plate draught ratio (3.0 and 12.44) at various plate draughts from 5 cm to 25 cm, at water depths from 62.5 cm to 77.5 cm and at wave lengths from 0.7 m to 2.1 m. The plate draught to wave length ratio $T/\lambda$ ranged from 0.03 to 0.3.

Waves were measured with resistance-type gauges. The data were accumulated by a computer and directly analysed using a method similar to that of Knott and Flower.[7] In this method, a pair of wave gauges on each side of the plates is sufficient to estimate the amplitude of incident, reflected, transmitted and beach-reflected waves. Measurements were also taken between the plates, but only for the case of $c/T = 12.44$, where the space was wide enough to avoid installing the gauges too close to the plates.

In order to reduce uncertainties due to measurement errors, four gauges were used on each side and three between the plates and the results which were obtained from various combinations of wave gauge pairs were averaged. The gauges in each group were placed at unequal spacings in an approximately 1 m long section along the channel.

Since measurement errors may be augmented when the distance between the two wave gauges is close to integer multiple of half the wave length, the wave gauge pairs having near-critical distances were discarded from the computations, thus reducing the possibility of substantial measurement errors.

The distances from the centres of the outer wave gauge groups to the plates were approximately 2 m.

Various wave lengths were employed with maximum values limited by approximately three times the water depth, to keep the waves in the deep water regime as much as possible.

*Results*

Transmission ($|T_c|$) and reflection ($|R_c|$) coefficients for the case $c/T = 3$ are plotted in Fig. 3. As expected, the system is close to resonance when the spacing is about half wave length ($c = 0.537\lambda$).

For the case $c/T = 12.44$, the transmission and reflection coefficients are given in Fig. 4, and the relative amplitudes of the waves propagating to the right ($|R_2|$) and to the left ($|L_1|$) in the space between the plates are given in Fig. 5.

From Figs. 3 and 4 it can be seen that for the transmission coefficient the experimental results are systematically smaller than the theoretical predictions, while for reflection the deviations of experimental data are quite small. This indicates that energy dissipation is involved mostly in the transmission mechanism, and not in the reflection.

The total energy loss by the bi-plate system, relative to the incident wave energy, is given by

$$\tilde{e}_T = 1 - (|\tilde{T}_c|^2 + |\tilde{R}_c|^2) \quad |\tilde{T}_c| = a_t/a_{i_0}, \ |\tilde{R}_c| = a_r/a_{i_0} \tag{10}$$

where $[\tilde{*}]$ denotes a measured quantity. The energy loss by the second plate only, relative to the incident wave energy, is given by

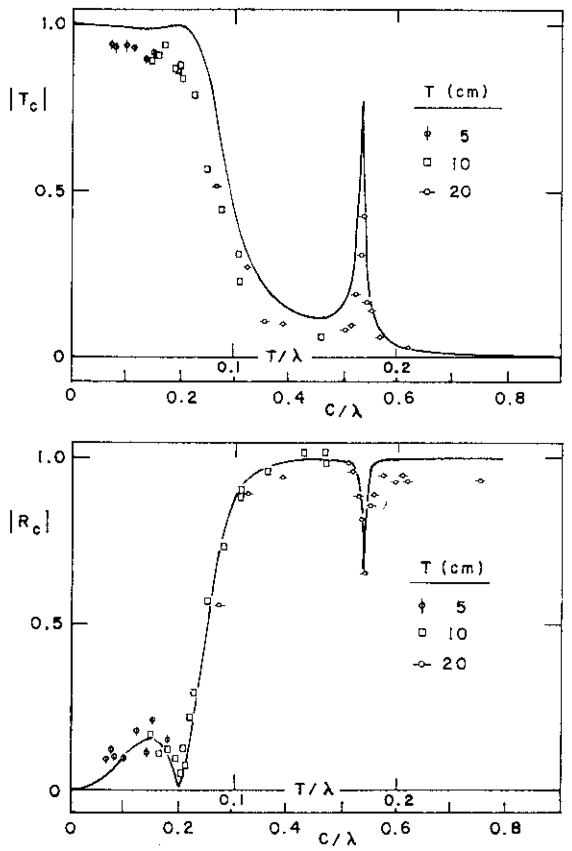


*Figure 3. The transmission ($|T_c|$) and reflection ($|R_c|$) coefficients for a bi-plate structure with $c/T = 3$. The theoretical results are given by the curves and the experimental results by the various symbols*

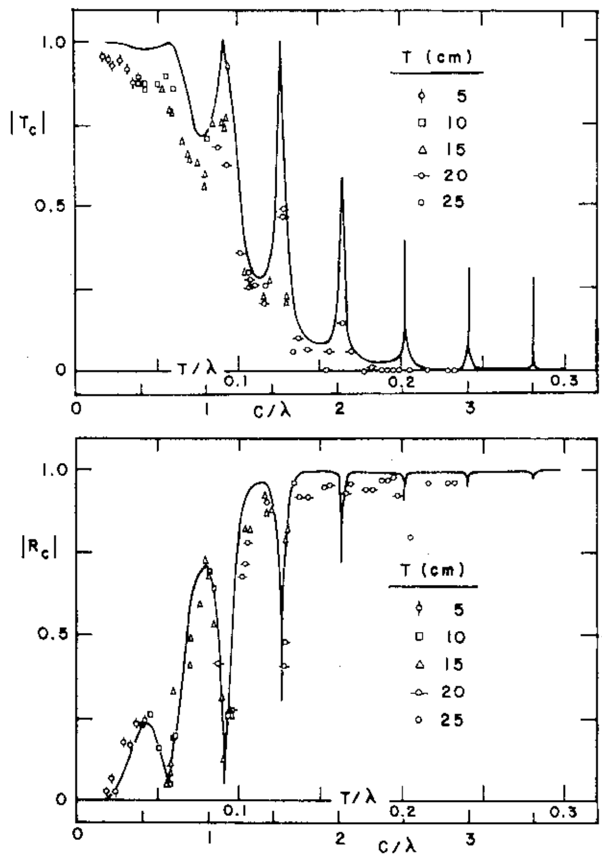


*Figure 4. As in Fig. 3, but with $c/T = 12.44$*

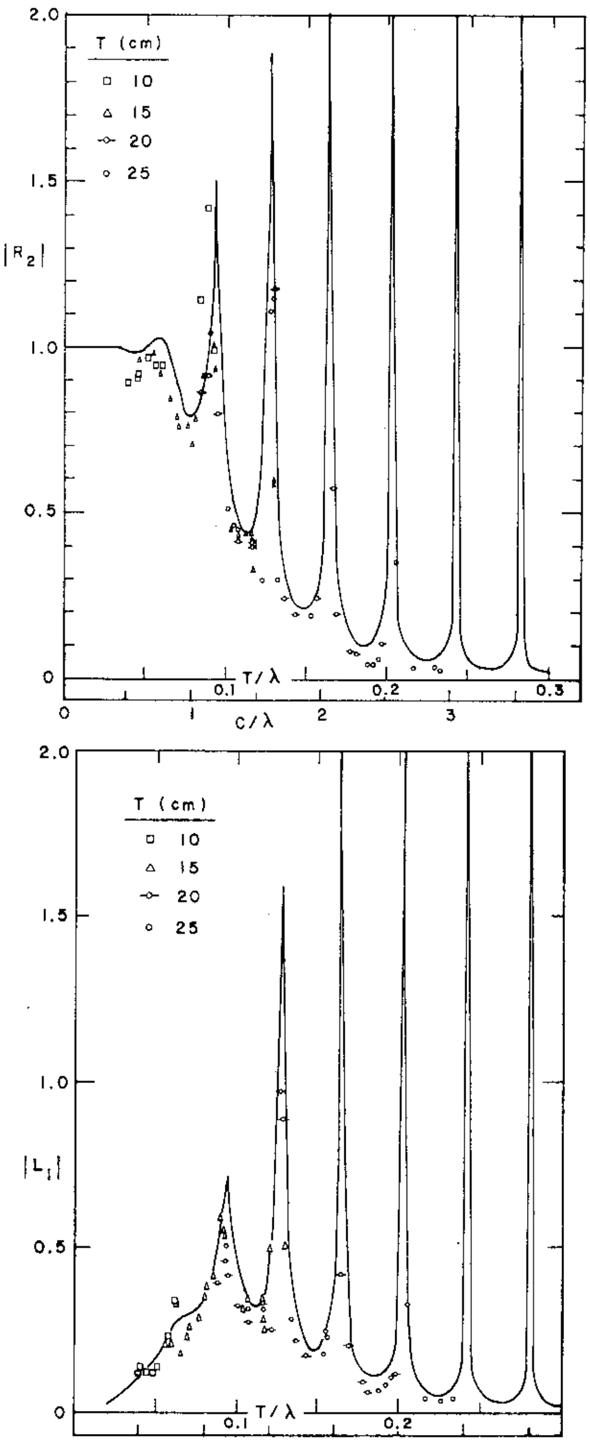


*Figure 5. The relative amplitudes of the waves in the space between the plates of a bi-plate structure with $c/T = 12.44$. (a) Waves propagating in the same direction as that of the incident-wave ($|R_2|$). (b) Waves propagating in a direction opposite to that of the incident-wave ($|L_1|$). The theoretical results are given by the curves and the experimental results by the various symbols*

$$\tilde{e}_2 = |\tilde{R}_2|^2 - (|\tilde{T}_c|^2 + |\tilde{L}_1|^2); \qquad |\tilde{R}_2| = a_{i_1}/a_{i_0}, \quad |\tilde{L}_1| = a_{r_1}/a_{i_0} \tag{11}$$

(see Fig. 2).

The experimental results for $\tilde{e}_T$ and $\tilde{e}_2$ are given in Fig. 6, for $c/T = 12.44$. It is seen that while the total

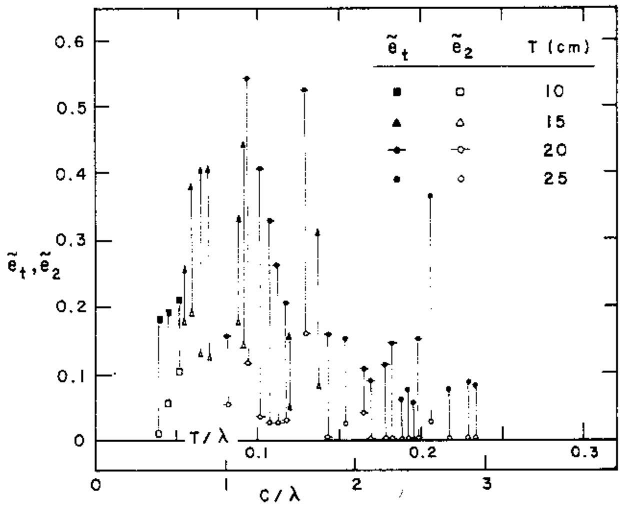


*Figure 6. Measured relative energy losses by a bi-plate structure with $c/T = 12.44$. Full symbols are for total energy loss ($\tilde{e}_T$) and hollow symbols for the loss on the second (down-stream) plate ($\tilde{e}_2$).*

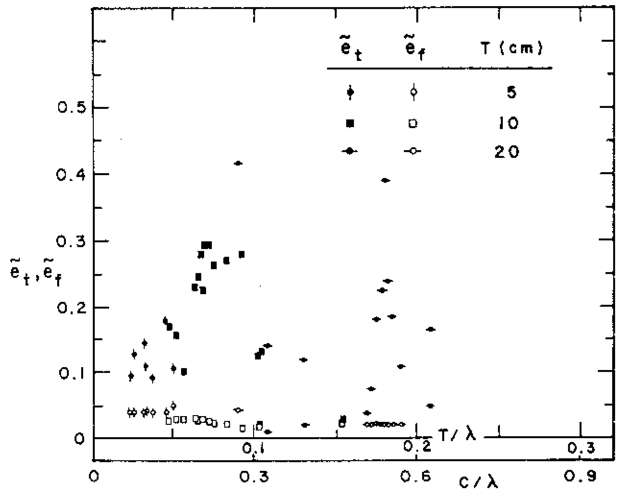


*Figure 7. Measured relative energy losses by a bi-plate structure with $c/T = 3$. Full symbols are for the total energy loss ($\tilde{e}_T$) and hollow symbols for the frictional loss ($\tilde{e}_f$)*

dissipation may reach as high as 50%, the dissipation at the second plate is usually below 18%; and in general, the difference between them, i.e. the dissipation at the first plate, is much greater than at the second plate. The dissipation at the first plate is given by

$$\tilde{e}_1 = \tilde{e}_T - \tilde{e}_2 = 1 + |\tilde{L}_1|^2 - (|\tilde{R}_2|^2 + |\tilde{R}_c|^2) \qquad (12)$$

The total relative measured energy dissipation, $\tilde{e}_T$ consists of dissipation due to vortex generation at the lower plate edges, and frictional losses in the channel. The frictional losses were estimated by measuring wave attenuation in obstacle-free channel.[6] By omitting them from the total measured dissipation it is possible to determine the experimental results of dissipation due to vortex shedding.

The total dissipation ($\tilde{e}_T$) and frictional ($\tilde{e}_f$) losses for $c/T = 3.0$ are shown in Fig. 7. It is seen that while the frictional losses seem to be small (about 2% to 4% of the incident energy), they are, for some values of $c/\lambda$, a significant portion of the total dissipation. Hence, in all of the following, the experimental results were corrected for frictional losses.

The incoming wave energy flux is given by

$$e_W = \frac{1}{4\sigma}\rho g^2 a_{i_0}^2 \qquad (13)$$

where $a_{i_0}$ is the incident wave amplitude (see Fig. 2). From equations (7), (8), (9) and (13), the relative vortex dissipation for the first plate (which is attacked by the waves from boh sides) is:

$$\frac{e_{v_1}}{e_W} = |1 - L_1|^{8/3}\frac{e_{v_0}}{e_W} = |1 - L_1|^{8/3}\frac{2.16(ka_{i_0})^{2/3}}{[kT(K_1^2 + \pi^2 I_1^2)]^{4/3}} \qquad (14)$$

and for the second plate (which is attacked only from one side):

$$\frac{e_{v_2}}{e_W} = |R_2|^{8/3}\frac{2.16(ka_{i_0})^{2/3}}{[kT(K_1^2 + \pi^2 I_1^2)]^{4/3}} \qquad (15)$$

Equations (14) and (15) describe a family of curves for various values of $a_i$ in the $e_v/e_w$-$T/\lambda$ plate. The graphical presentation is reduced to a single curve by showing $(e_v/e_w)/(ka_i)^{2/3}$ as a function of $T/\lambda$.

The experimental results were obtained by substituting the measured quantities $|\tilde{T}_c|$, $|\tilde{R}_c|$, $|\tilde{R}_2|$, $|\tilde{L}_1|$ (after correction for friction) in equations (11) and (12). Dividing these equations by $(ka_i)^{2/3}$ yields, for the first plate:

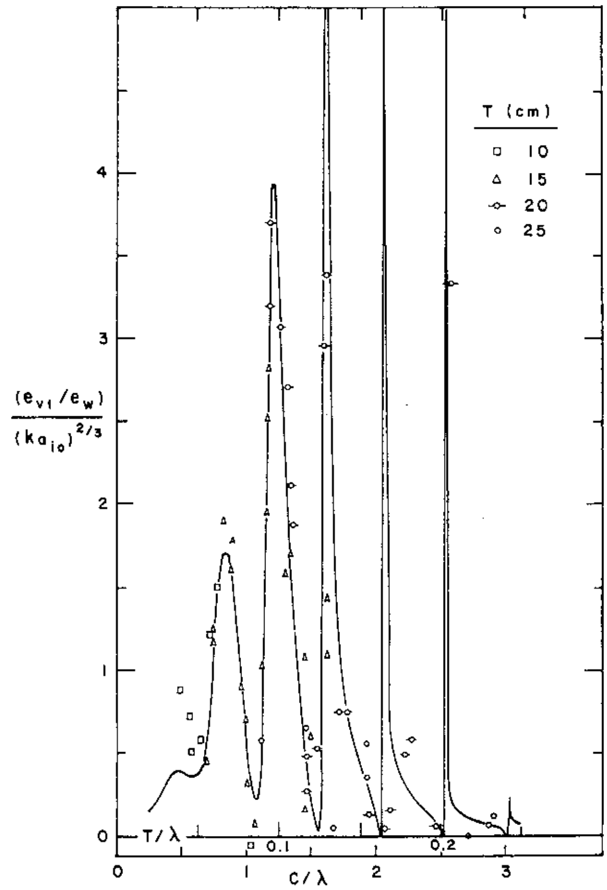


*Figure 8. Energy dissipation due to vortex generation by the first (up-stream) plate of a bi-plate structure with $c/T = 12.44$. The theoretical results are given by the curve and the experimental results by the various symbols*

$$\frac{\tilde{e}_{v_1}}{(k\tilde{a}_{i_0})^{2/3}} = \frac{1+|\tilde{L}_1|^2-(|\tilde{R}_2|^2+|\tilde{R}_c|^2)}{(k\tilde{a}_{i_0})^{2/3}} \tag{16}$$

and for the second:

$$\frac{\tilde{e}_{v_2}}{(k\tilde{a}_{i_1})^{2/3}} = \frac{|\tilde{R}_2|^2-(|\tilde{T}_c|^2+|\tilde{L}_1|^2)}{(k\tilde{a}_{i_1})^{2/3}} \tag{17}$$

The experimental (equations (16) and (17)) and theoretical (equations (14) and (15)) results for the first and second plates for $c/T = 12.44$ are given in Figs. 8 and 9 respectively.

For the first plate (Fig. 8, equation (16)), the agreement between theory and experiments is very good. For the second plate (Fig. 9, equation (17)), there is a fairly good agreement, but the scatter of experimental data points is quite large. This is possible due to the fact that the errors in measuring $a_{i_1}$ between the plates ($a_{i_1}$ is used in equation (17)), are greater than the errors in measuring $a_{i_0}$ in the outer region ($a_{i_0}$ is used in equation (16)).

Since for $c/T = 3.0$ waves were not measured between the plates, it was impossible to compare experiments with theory for the individual plates. In this case, comparison was done only for the entire bi-plate system. The total theoretical energy loss is given by the sum of equations (14) and (15).

The experimental result is given by equation (10). Correcting the experiments for frictional losses and dividing both equations (10) and the sum of equations (14) and (15) by $(ka_{i_0})^{2/3}$ provides the comparison between theory and experiments, which is presented in Fig. 10. We find the agreement in this figure remarkable.

The generally good agreement found in the study confirms the presented theory.

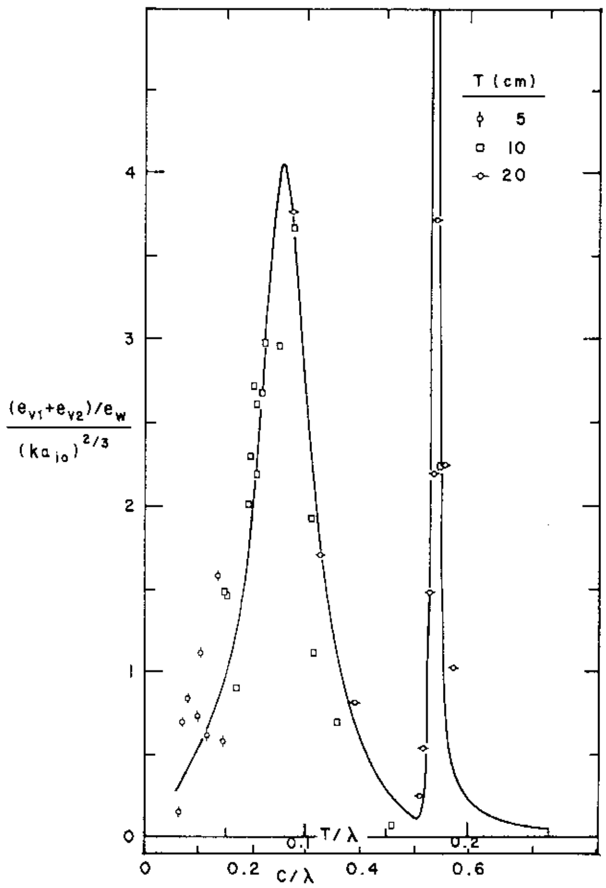


*Figure 10. Energy dissipation due to vortex generation by a bi-plate structure with $c/T = 3$*

## 4. CONCLUSIONS

The wide spacing approximation method was tested experimentally and was found adequate in the ranges: $0.3 < \lambda/c < 4$ (for $c/T = 12.44$), and $1.3 < \lambda/c < 16.7$ (for $c/T = 3$). The union of these two ranges ($0.3 < \lambda/c < 16.7$) is much greater than the assumed requirement $\lambda/c < 1$. The origin of the term 'wide spacing' seems to lie in the assumption $\lambda/c = o(1)$. Our experiments, as well as earlier results by Ohkusu,[4] indicate that the method is effective even for $\lambda/c$ as high as 17. Thus, the name 'wide spacing' should be thought of as referring to the fact that $T/c = o(1)$ (0.08 and 0.33 in our experiments), rather than $\lambda/c = o(1)$.

The experimental results reported in this paper indicate that our single plate vortex dissipation model works very well for bi-plate configurations. This conclusive result makes our vortex dissipation model a reliable tool for calculation of the energy losses in multi-plate structures. Note that the generalisation of this model for plates which are free to roll and/or sway is rather straightforward.

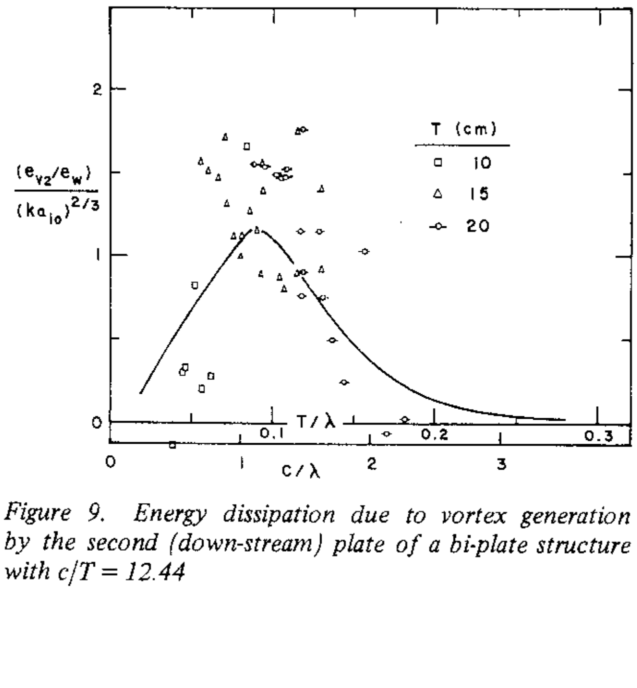


*Figure 9. Energy dissipation due to vortex generation by the second (down-stream) plate of a bi-plate structure with $c/T = 12.44$*

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